1. $\boxed{\text{DEFINITION:}}$ **Two-Sided Limit**

Notation: $\lim\limits_{x \to a} f(x) = L$

Words: the limit of f(x), as x approaches a, is L.

It means: As the x-values get closer + closer to a (larger + smaller than a) the y-values of f(x) get close to L. In fact, the y's can be forced arbitrarily close to L.

Evaluate the limits below numerically. Estimate the limit to 4 decimal places, if possible.

2. $\lim\limits_{x\to 0} \frac{\sin(x)}{x} = \boxed{1}$

x	sin(x)/x
0.1	0.99833
0.01	0.99998
0.001	0.99999
0	DNE
−0.001	0.99999
−0.01	0.99998
−0.1	0.99833

3. $\lim\limits_{x\to 2} \frac{\cos(x)(x-2)}{3x^2-5x-2} = \boxed{0.0594}$

x	$\cos(x)(x-2)/(3x^2-5x-2)$
2.1	−0.069157
2.01	−0.060486
2.001	−0.05955
2.00001	−0.05945
2	
1.99999	−0.059448
1.999	−0.059345
1.99	−0.058397
1.9	−0.04825

4. $\lim\limits_{x\to -1} \frac{|x+1|}{x+1} = \boxed{\text{DNE}}$

x	$\lvert x+1\rvert/(x+1)$
−0.9	1
−0.99	1
−0.999	1
−0.9999	1
−1	DNE
−1.0001	−1
−1.001	−1
−1.01	−1
−1.1	−1

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$\lim\limits_{x\to -1^+} \frac{|x+1|}{x+1} = \boxed{1}$

$\lim\limits_{x\to -1^-} \frac{|x+1|}{x+1} = \boxed{-1}$

5. $\lim\limits_{x\to 1} \frac{1}{x-1} = \boxed{\text{DNE}}$

x	1/(x−1)
1.1	10
1.01	100
1.001	1000
1.0001	10,000
1	DNE
0.9999	−10,000
0.999	−1000
0.99	−100
0.9	−10

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$\lim\limits_{x\to 1^+} \frac{1}{x-1} = +\infty$

$\lim\limits_{x\to 1^-} \frac{1}{x-1} = -\infty$

6. $\boxed{\text{DEFINITION:}}$ **One-Sided Limits**

Notation:

$\lim_{x \to a^-} f(x) = L$

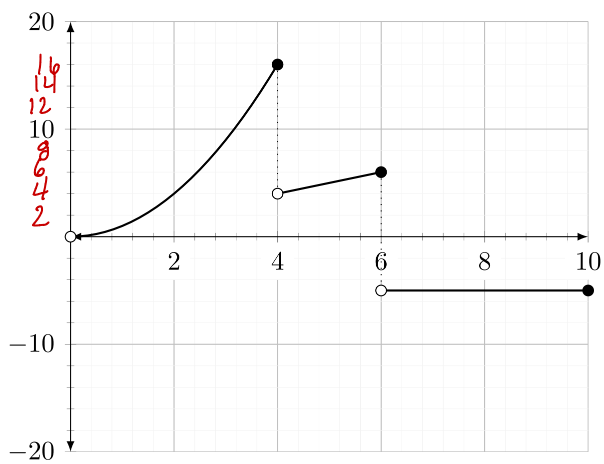
x-values approach a only on left or below or x-values less than x=a

$\lim_{x \to a^+} f(x) = L$

x-values approach x=a only on the right or above or from x-values larger than x=a.

Limits can also be evaluated graphically.

7. The function $g(x)$ is graphed below. Use the graph to fill in the blanks.



(a) $\lim_{x \to 4^-} g(x) =$ 16

(b) $\lim_{x \to 4^+} g(x) =$ 4

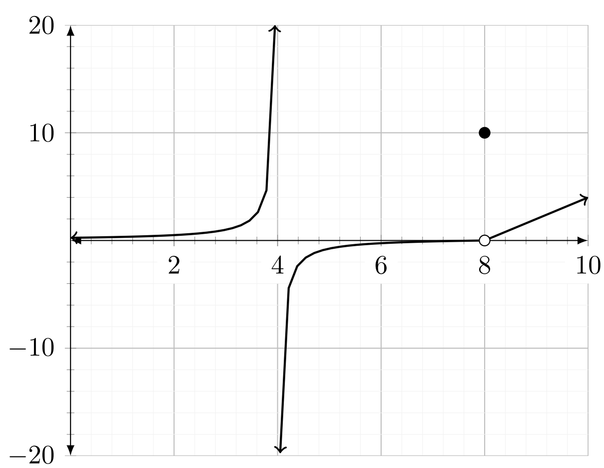
(c) $\lim_{x \to 4} g(x) =$ DNE

(d) $g(4) =$ 16

(e) $\lim_{x \to 8} g(x) =$ −5

(f) $g(8) =$ −5

8. The function $h(x)$ is graphed below. Use the graph to fill in the blanks.



(a) $\lim_{x \to 4^-} h(x) =$ $+\infty$

(b) $\lim_{x \to 4^+} h(x) =$ $-\infty$

(c) $\lim_{x \to 4} h(x) =$ DNE

(d) $h(4) =$ DNE

(e) $\lim_{x \to 8} h(x) =$ 0

(f) $h(8) =$ 10

9. Find any vertical asymptotes of $f(x) = \frac{2}{x+5}$ and *justify* your answer using a limit.

V.a.: $x = -5$ ← Value makes denominator zero.

Justification:

$$\lim_{x \to -5^+} \frac{2}{x+5} = +\infty$$

as $x \to -5^+$ (#'s like $-4.9, -4.99$)

$x + 5 \to 0^+$

10. Sketch the graph of an function that satisfies *all* of the given conditions. Compare your answer with that of your neighbor.

$\lim_{x\to 0^-} f(x) = 1 \qquad \lim_{x\to 0^+} f(x) = -2 \qquad \lim_{x\to 4^-} f(x) = 3 \qquad \lim_{x\to 4^+} f(x) = 0$

$f(0) = -2 \qquad\qquad f(4) = 1$