

SECTION 3-2: THE DERIVATIVE AS A FUNCTION

Read Section 3.2. Work the embedded problems.

1. Definition of the Derivative Function

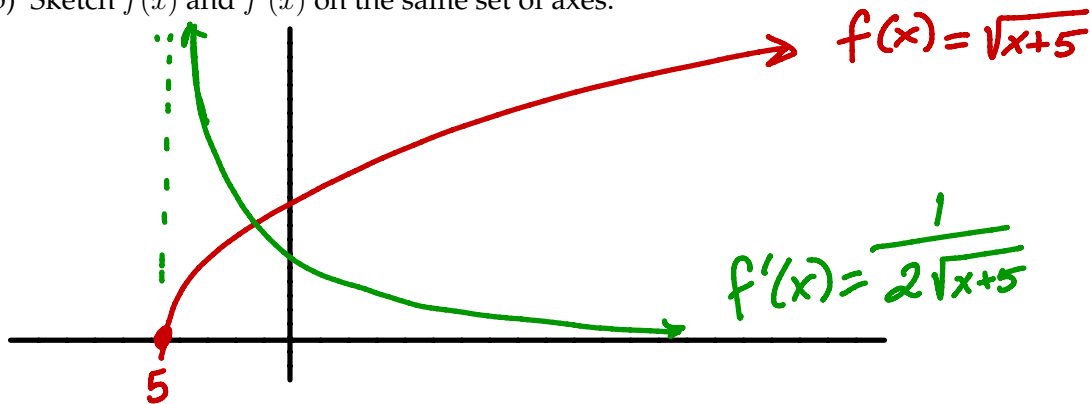
$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

2. Let $f(x) = \sqrt{x+5}$.

(a) Use the definition of the derivative to find $f'(x)$.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \left(\frac{\sqrt{x+h+5} - \sqrt{x+5}}{h} \right) \left(\frac{\sqrt{x+h+5} + \sqrt{x+5}}{\sqrt{x+h+5} + \sqrt{x+5}} \right) = \lim_{h \rightarrow 0} \frac{x+h+5 - (x+5)}{h(\sqrt{x+h+5} + \sqrt{x+5})} \\ &= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h+5} + \sqrt{x+5})} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h+5} + \sqrt{x+5}} = \frac{1}{\sqrt{x+0+5} + \sqrt{x+5}} = \frac{1}{2\sqrt{x+5}} \end{aligned}$$

(b) Sketch $f(x)$ and $f'(x)$ on the same set of axes.



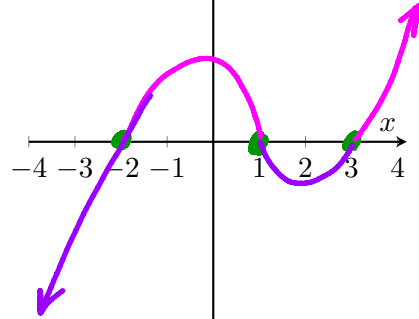
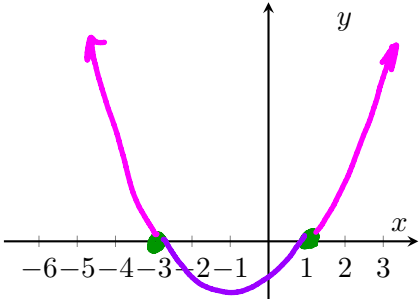
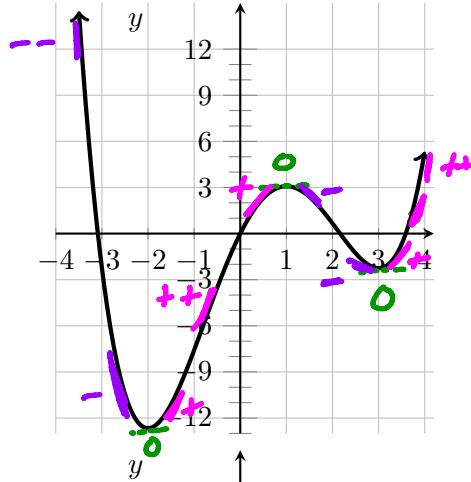
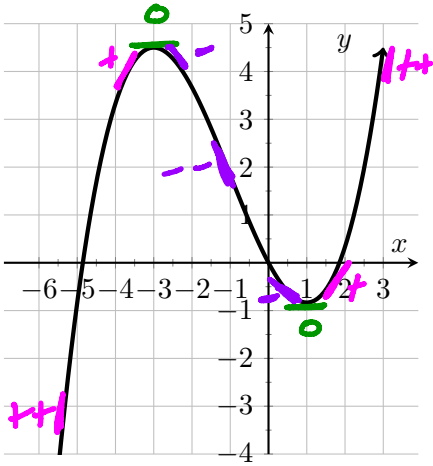
(c) Write the equation of the line tangent to $f(x)$ at $x = 0$.

need point $P(0, f(0)) = (0, \sqrt{5})$ and slope $m = f'(0) = \frac{1}{2\sqrt{5}}$

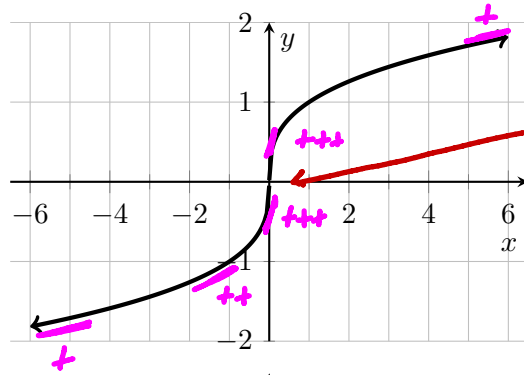
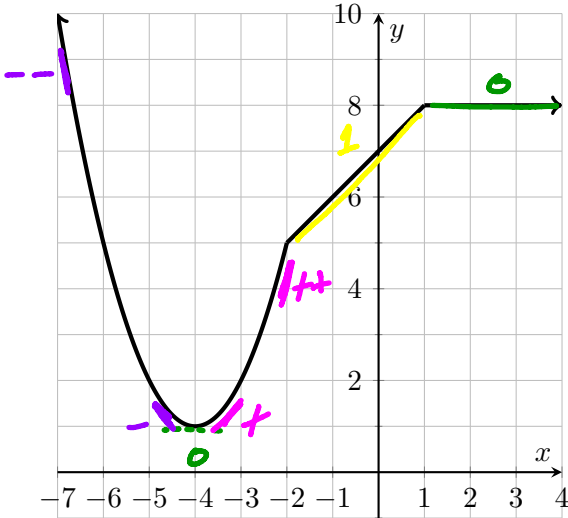
line: $y - \sqrt{5} = \frac{1}{2\sqrt{5}}(x - 0)$ or

$$y = \frac{x}{2\sqrt{5}} + \sqrt{5}$$

3. For each graph below, sketch the graph of $f'(x)$ on the axes below.



← Where does this function fail to be differentiable? $x = -2, x = 1$.



right at $x = 0$, the tangent is vertical!

