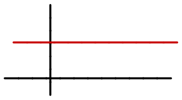
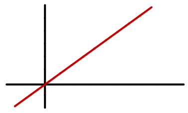
 

makes steeper $y=x$

shifts up 1

1. Using what you know about the graphs of the functions below, determine their derivatives

$f(x) = 10$ $\quad g(x) = x^1$ $\quad h(x) = \pi x$ $\quad j(x) = \pi x + 1$

$f'(x) = $ <u>0</u> $\quad g'(x) = $ <u>1</u> $\quad h'(x) = $ <u>π</u> $\quad j'(x) = $ <u>π</u>

2. Use the definition of the derivative to find the derivatives for each of the following functions:

(a) $f(x) = x^2$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h} = \lim_{h\to 0} \frac{2xh+h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(2x+h)}{h} = \lim_{h\to 0} 2x+h = \boxed{2x}$$

$$\boxed{f(x) = x^2, \quad f'(x) = 2x^1}$$

(b) $f(x) = x^3$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$

$$= \lim_{h\to 0} \frac{h(3x^2+3xh+h^2)}{h} = \lim_{h\to 0} 3x^2+3xh+h^2 = 3x^2$$

$$\frac{d}{dx}\left[x^3\right] = 3x^2$$

or slick new notation!

$$\boxed{f(x) = x^3, \quad f'(x) = 3x^2}$$

pattern?

- exponent ↓ by 1
- constants hang around

$$\frac{d}{dx}\left[x^{-1}\right] = -1x^{-2} = -x^{-2}$$

3. Recall the following results below:

work	$f(x)$	$f'(x)$
(worksheet §3.1)	x^{-1}	$-1x^{-2}$
(§3.1 # 20)	$3x^{-2}$	$-6x^{-3}$ $= 3(-2)x^{-3}$
(§3.2 #59)	$(\sqrt{2})x^{1/2}$	$\frac{\sqrt{2}}{2}x^{-1/2}$ $= \sqrt{2}\left(\frac{1}{2}\right)x^{-1/2}$

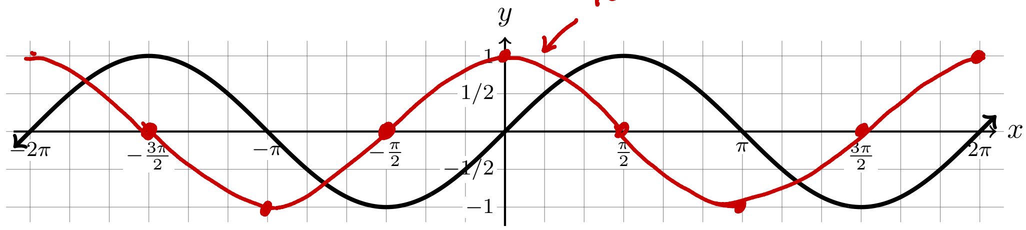
4. Use the data above to fill in the rules below. Assume c and n are fixed numbers.

$\frac{d}{dx}[c] = $ <u>0</u> $\quad \frac{d}{dx}[x^n] = $ <u>nx^{n-1}</u> $\quad \frac{d}{dx}[x^n + c] = $ <u>nx^{n-1}</u> $\quad \frac{d}{dx}[cx^n] = $ <u>$c \cdot nx^{n-1}$</u>

5. Use the graphs of $f(x) = \sin(x)$ and $g(x) = \cos(x)$ (below) to sketch the graph of their derivatives $f'(x)$ and $g'(x)$.

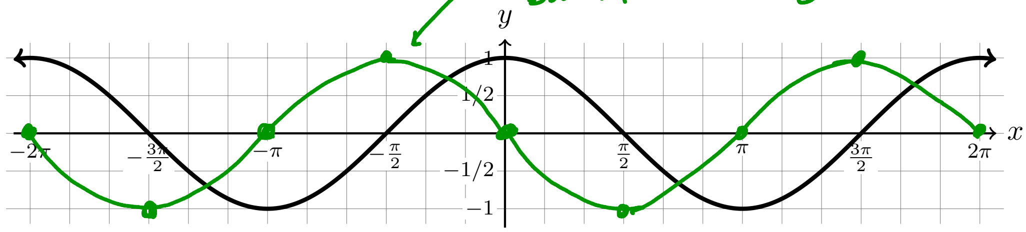
$f(x) = \sin(x)$

looks like $y = \cos(x)$



$g(x) = \cos(x)$

like sine but reflected ... $y = -\sin(x)$



6. Base on the work above, guess answers: $\frac{d}{dx}[\sin(x)] = \underline{\cos(x)}$ $\frac{d}{dx}[\cos(x)] = \underline{-\sin(x)}$

7. Four Big Rules

(a) Constant Multiple

$$\frac{d}{dx}[c\,f(x)] = c\frac{d}{dx}[f(x)]$$

eg:

$y = 50\sin(x)$

$y' = 50\cos(x)$

or

$\frac{d}{dx}[50\sin(x)]$

$= 50\frac{d}{dx}[\sin(x)]$

$= 50\cos(x)$

(b) Sum (and Difference)

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$

- $\frac{d}{dx}[x^2+\cos x] = \frac{d}{dx}[x^2] + \frac{d}{dx}[\cos x]$
  $= x^2 - \sin x$
- If $y = x^2+\cos(x)$, then $y' = 2x - \sin(x)$

(c) Product

$$\frac{d}{dx}[f(x)\,g(x)] = \frac{d}{dx}[f(x)]\cdot g(x) + f(x)\cdot\frac{d}{dx}[g(x)]$$

$$\frac{d}{dx}[x^2\sin(x)] = 2x\cdot\sin(x) + x^2\cdot\cos x$$

(d) Quotient

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)]\,g(x) - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

- $\frac{d}{dx}\left[\frac{x^2}{\sin x}\right]$

$$= \frac{(2x)(\sin x) - (x^2)(\cos(x))}{(\sin x)^2}$$