

SECTION 3-5: DERIVATIVES OF TRIGONOMETRIC FUNCTIONS (EXTRA PRACTICE)

1. (Revisit the spring problem:) A mass on a spring vibrates horizontally on a smooth level surface. Its equation of motion is $x(t) = 8 \sin(t)$, where t is in seconds and x is in centimeters.

(a) We found:

$$v(t) = x'(t) = 8 \cos(t) \text{ and } a(t) = v'(t) = x''(t) = -8 \sin(t)$$

(b) We found:

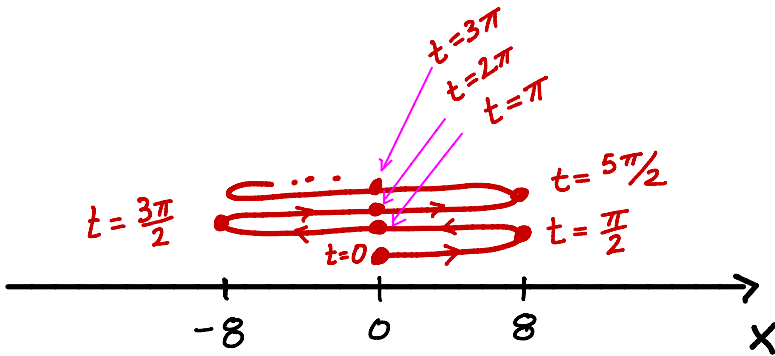
$$x(2\pi/3) = 4\sqrt{3} \text{ cm}$$

$$x'(2\pi/3) = -4 \text{ cm/s}$$

$$x''(2\pi/3) = -4\sqrt{3} \text{ cm/s}^2$$

At $t = 2\pi/3$, the mass is moving to the left and slowing down.

(c) Draw a picture of the motion of the mass and include the time(s) at which the mass changes direction.



quick check at
 $t=0$:
 $x(0) = 0$ ← initial position
 $x'(0) = 8$ ← moving to the right

The mass turns around when $v(t) = x'(t) = 0$ or $8 \cos(t) = 0$. We know $\cos(t) = 0$ when $t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$

2. Higher Order Derivatives. For each function below, find $f'(x)$, $f''(x)$, $f'''(x)$, $f^{(4)}(x)$, $f^{(82)}(x)$

(a) $f(x) = x^5 + 2x^2 + 1$

$$f'(x) = 5x^4 + 4x$$

$$f''(x) = 20x^3 + 4$$

$$f'''(x) = 60x^2 \quad f^{(82)}(x) = 0$$

$$f^{(4)}(x) = 120x$$

(b) $f(x) = 2 \sin(x)$

$$f'(x) = 2 \cos(x)$$

$$f''(x) = -2 \sin(x)$$

$$f'''(x) = -2 \cos(x)$$

$$f^{(4)}(x) = 2 \sin(x) = f(x) \leftarrow \text{We're back where we started!}$$

$$\text{So } f^{(80)}(x) = 2 \sin(x).$$

$$\text{So } f^{(82)}(x) = f''(x) = -2 \sin(x)$$

3. Other ways of denoting derivatives.

$$y = f(x)$$

- $f'(x), f''(x), f'''(x), f^{(4)}(x)$

- $y', y'', y''', y^{(4)}$

- $\frac{d}{dx} [f(x)], \frac{d^2}{dx^2} [f(x)], \frac{d^3}{dx^3} [f(x)]$

- $\frac{df}{dx}, \frac{d^2 f}{dx^2}, \frac{d^3 f}{dx^3}, \dots$

- $\frac{dy}{dx}, \frac{d^2 y}{dx^2}, \frac{d^3 y}{dx^3}, \dots$