1. Recall Two Versions of the Chain Rule

$\boxed{A}$ $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$

$\boxed{B}$ $y = f(u)$, $u = g(x)$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

2. Understanding what the "formulas" in the book are trying to communicate:

In §3.5

$\frac{d}{dx}\left[\sec(x)\right] = \sec(x)\tan(x)$

In §3.6

- $\frac{d}{dx}\left[\sec(g(x))\right] = \left[\sec(g(x))\tan(g(x))\right] \cdot g'(x)$
- $\frac{d}{dx}\left[\sec(u)\right] = \left[\sec(u)\tan(u)\right] \cdot \frac{du}{dx}$

3. Find the derivatives.

(a) $g(\theta) = \sqrt[5]{\sin(\frac{\theta}{\pi})} = \left[\sin\left(\frac{1}{\pi}\theta\right)\right]^{\frac{1}{5}}$

$$g'(\theta) = \frac{1}{5} \cdot \left[\sin\left(\frac{1}{\pi}\theta\right)\right]^{-\frac{4}{5}} \cdot \cos\left(\frac{1}{\pi}\theta\right) \cdot \frac{1}{\pi} = \frac{\cos\left(\frac{1}{\pi}\theta\right)}{5\pi\left(\sin\left(\frac{1}{\pi}\theta\right)\right)^{4/5}}$$

(b) $f(x) = (\sec(3x) + \csc(2x))^5$

$$f'(x) = 5\left(\sec(3x) + \csc(2x)\right)^4 \cdot \left(3\sec(3x)\tan(3x) - 2\csc(2x)\cot(2x)\right)$$

(c) $g(x) = \frac{\cos(x^2+1)}{x^3+1}$

a holding pattern

$$g'(x) = \frac{\boxed{\frac{d}{dx}\left[\cos(x^2+1)\right]}(x^3+1) - \cos(x^2+1)\cdot(3x^2)}{(x^3+1)^2}$$

$$= \frac{-2x\sin(x^2+1)(x^3+1) - 3x^2\cos(x^2+1)}{(x^3+1)^2}$$

(d) $h(x) = \overset{f}{(2x-1)^3}\overset{g}{(2x+1)^5}$

$$h'(x) = \underbrace{3(2x-1)^2(2)}_{f'}\underbrace{(2x+1)^5}_{g} + \underbrace{(2x-1)^3}_{f}\cdot\underbrace{5(2x+1)^4(2)}_{g'}$$

$$= 6(2x-1)^2(2x+1)^5 + 10(2x-1)^3(2x+1)^4$$

4. Find all x-values where the tangent to $f(x) = \frac{5}{(8x-x^2)^3}$ is horizontal.

$$f(x) = 5(8x-x^2)^{-3}$$

$$f'(x) = 5(-3)(8x-x^2)^{-4}(8-2x) = \frac{-15(8-2x)}{(8x-x^2)^4} = 0$$

So $8-2x=0$.

So $\boxed{x=4}$

5. Find all x-values where the tangent to $f(x) = (4-x)^3$ is parallel to $y + 6x = 8$.

$\Rightarrow y = -6x+8$. So $\underline{\underline{m=-6}}$

$$f'(x) = 3(4-x)^2(-1) = -6$$

So $(4-x)^2 = 2$

$$4-x = \pm\sqrt{2}$$

$$\boxed{x = 4 \pm \sqrt{2}}$$