1. Recall the definition of the derivative:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad ; \quad \text{So } f'(a) \approx \frac{f(a+h)-f(a)}{h}$$

for $h = 0.001$ ← close to 0

2. Let $f(x) = e^x$. **Estimate** $f'(x)$ (a.k.a. the slope of the tangent line) using the limit definition for each of the values below. (Use a calculator!)

Pattern?

$f(a) = f'(a)$?!!

y-values = derivative values?!

(a) $f'(0)$ $\approx \dfrac{e^{0.001} - e^{0}}{0.001 - 0} = 1.0005 \approx f(0) = 1$ ← Note.

$* \; f(0) = 1$

(b) $f'(1)$ $\approx \dfrac{e^{1.001} - e^{1}}{1.001 - 1} = 2.71964 \approx f(1) = e^1 = e = 2.7182$

(c) $f'(2)$ $\approx \dfrac{e^{2.001} - e^{2}}{2.001 - 2} = 7.39275 \approx f(2) = e^2 = 7.38905$

(d) $f'(-1)$ $\approx \dfrac{e^{-1.001} - e^{-1}}{-1.001 - (-1)} = 0.36769 \approx f(-1) = e^{-1} = 0.36787$

3. Derivative Rules for Exponential Functions

$$\frac{d}{dx}\left[e^x\right] = e^x$$

Again, y-values equal derivative values

$$\frac{d}{dx}\left[a^x\right] = (\ln a)a^x$$

Note: $a^x = e^{(\ln a)x}$.

Do you see the relationship?

4. Examples: Find the derivatives.

(a) $y = x^4 e^x$ $\quad f \quad g$

$$y' = 4xe^x + x^4 e^x$$
$$f' \cdot g + f \cdot g'$$

(b) $y = e^{x^2} = e^{(x^2)}$ ← chain rule!

$$y' = (e^{x^2})(2x) = 2xe^{x^2}$$

(c) $y = 5^{-x} = 5^{(-x)}$ ← chain rule!

$$y' = ((\ln 5)5^{-x})(-1)$$
$$= (-\ln 5)5^{-x}$$

Alternate:
$$y = \left(\tfrac{1}{5}\right)^x$$
$$y' = \ln\left(\tfrac{1}{5}\right)\left(\tfrac{1}{5}\right)^x$$

(d) $f(x) = x^5 + 5^x$

$$f'(x) = \underbrace{5x^4}_{\text{power rule}} + \underbrace{(\ln 5)5^x}_{\text{exponential rule}}$$

Do you see why different rules are used?

5. A population of bacteria is modeled by the equation $P(t) = 100e^{0.04t}$ where P is the number of bacterial and t is measured in hours.

(a) Find $P(0)$, $P(1)$, and $P(100)$. Give units with your answers. What do these numbers represent?

$P(0) = 100$ bacteria

$P(1) = 104.08$ bacteria

$P(100) = 5459.8$ bacteria

Tell us how large the population of bacteria is at each hour.

(b) Find $P'(0)$, $P'(1)$, and $P'(100)$. Give units with your answers. What do these numbers represent?

$$P'(t) = 100 \cdot e^{0.04t}(0.04)$$
$$= 4e^{0.04t}$$

$P'(0) = 4$

$P'(1) = 4.163$

$P'(100) = 218.39$

units: bactera/hour

Tells us the rate of change of the population.

(c) Find $P'(0)/P(0)$, $P'(1)/P(1)$ and $P'(100)/P(100)$. What do these numbers represent?

$$\frac{P'(0)}{P(0)} = \frac{P'(1)}{P(1)} = \frac{P'(100)}{P(100)} = 0.04;$$

This is the percent change in population.

6. Let $P(t) = P_0 e^{kt}$. Find $P'(t)/P(t)$ and use this to explain what k represents.

$$P'(t) = kP_0 e^{kt}$$

So
$$\frac{P'}{P} = \frac{kP_0 e^{kt}}{P_0 e^{kt}} = k.$$

So k gives the percent change in population.