1. The Extreme Value Theorem

If $f(x)$ is continuous on a closed bounded interval $[a,b]$, then $f(x)$ has an absolute minimum and an absolute maximum on $[a,b]$.

How to use this Theorem?

* ① Make sure it applies!
* ② Find all crit. pts.
* ③ Check y-values at crit. pts and endpoints.

2. For each problem below, (i) find all critical points of the function on the given interval, (ii) use the Extreme Value Theorem to determine the absolute maximum and absolute minimum of the function, and (iii) use technology to graph the function on the interval to confirm your answer.

   (a) $f(x) = 3x^{1/3} - x$ on $[-1, 8]$

① $f(x)$ is continuous on $[-1, 8]$

② Find crit. pts.

$$f'(x) = 3 \cdot \frac{1}{3} x^{-2/3} - 1 = \frac{1}{x^{2/3}} - 1$$

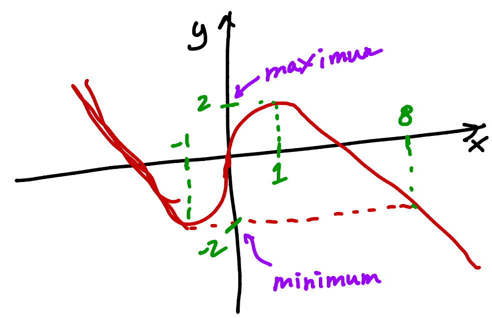
f' undefined when $x = 0$ ← what is happening here?

$f' = 0$ when $\frac{1}{x^{2/3}} - 1 = 0$ or $x^{2/3} = 1$

So $x = \pm 1$

crit. pts: $x = -1, 0, 1$

graph



③

x	-1	0	1	8
$f(x) = y$	$3(-1)^{1/3} - (-1)$ $= -3+1$ $= -2$	0	$3-1$ $= 2$	$3(8)^{1/3} - 8$ $= 3 \cdot 2 - 8$ $= -2$

Conclusions: on $[-1, 8]$

$f(x)$ has a absolute maximum of $y = 2$ at $x = 1$,

an absolute minimum of $y = -2$ at

$x = -1$ and $x = -2$

(b) $f(x) = \cos(x) - \frac{x}{2}$ on $[0, 2\pi]$

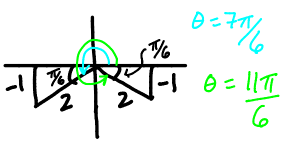
f is continuous everywhere.

$f'(x) = -\sin(x) - \frac{1}{2}$

f' is never undefined

$f' = 0$ when $-\sin(x) - \frac{1}{2} = 0$

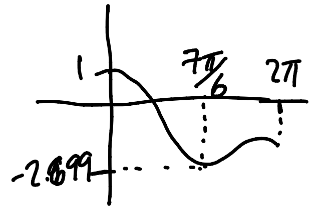
or $\sin(x) = -\frac{1}{2}$.



crit. pts $x = 7\pi/6$

and $x = \frac{11\pi}{6}$

x	0	2π	$\frac{7\pi}{6}$	$\frac{11\pi}{6}$
y	1	$1-\pi$ ≈ -2.14159	$\cos\left(\frac{7\pi}{6}\right) - \frac{7\pi}{12} = \frac{-\sqrt{3}}{2} - \frac{7\pi}{12} = -2.699$	$\cos\left(\frac{11\pi}{6}\right) - \frac{11\pi}{6} = \frac{\sqrt{3}}{2} - \frac{11\pi}{6} \approx -2.014$



maximum is $y = 1$

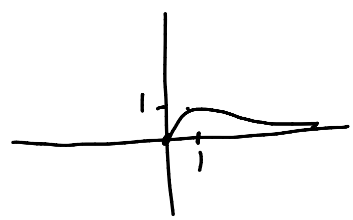
minimum is $y = -2.699$

(c) $g(x) = \frac{2x}{x^2+1}$ on $[0, 10]$

g is continuous everywhere.

max : $y=1$ at $x=1$ min : 0 at $x=0$

$$g'(x) = \frac{(x^2+1)(2) - 2x(2x)}{(x^2+1)^2} = \frac{2-2x^2}{(x^2+1)^2}$$



crit. pts $x = \pm 1$. only $+1$ interval.

Ans: $x=1$ crit. number.

x	0	1	10
y	0	1	$\frac{20}{101}$

3. (Bonus Problem) An object with a weight of W is dragged along a horizontal plan by a force acting along a rope attached to the object. If the rope makes an angle of θ with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin\theta + \cos\theta} = \mu W (\mu \sin\theta + \cos\theta)^{-1}$$

where μ is a positive constant called the coefficient of friction. Assume $0 \leq \theta \leq \pi/2$. Show that F is minimized when $\tan\theta = \mu$.

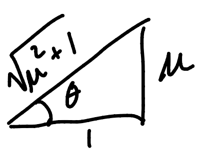
$$\frac{dF}{d\theta} = uW(-1)(\mu \sin\theta + \cos\theta)^{-2}(\mu\cos\theta - \sin\theta)$$

$F(0) = \mu W$

$F(\pi/2) = W$

$\frac{dF}{d\theta} = 0$ when $\mu\cos\theta - \sin\theta = 0$ or $\boxed{\mu = \tan\theta}$

$$F(\arctan\mu) = \frac{\mu W}{\frac{\mu \cdot \mu}{\sqrt{\mu^2+1}} + \frac{1}{\sqrt{\mu^2+1}}}$$



$$= \frac{\mu W}{\sqrt{\mu^2+1}}$$ ← Less than μW and W