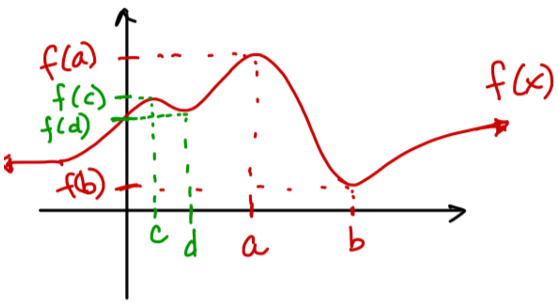
SECTION 4.3: MAXIMUMS AND MINIMUMS

- local and absolute maximums and minimums: what they are and how to find them
- critical points
- closed-interval method

1. local and absolute maximums and minimums: what they are



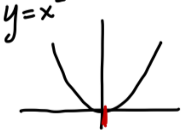
- $f(a)$ is an absolute maximum because $f(a) \geq f(x)$ for all x in domain
- $f(b)$ is an absolute minimum because $f(b) \leq f(x)$ for all x in domain.
- $f(c)$ is a local maximum because $f(c) \geq f(x)$ for all x in an open interval around c.
- $f(d)$ is a local minimum because $f(d) \leq f(x)$ for all x in an open interval around d.

Note: maximums + minimums are y-values.

- critical pts

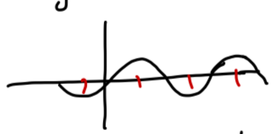
2. A variety of examples

$y=x^2$



- one absolute min. $(y=0)$
- no abs/loc max

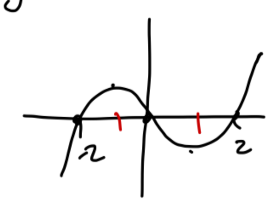
$y=\sin(x)$



- one abs max: $y=1$
- one abs min: $y=-1$

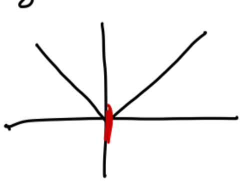
They occur at an infinite # of places

$y=(x+2)(x)(x-2)$



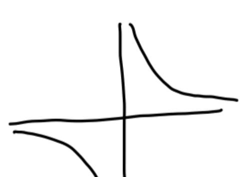
one local max, one local min

$y=|x|$



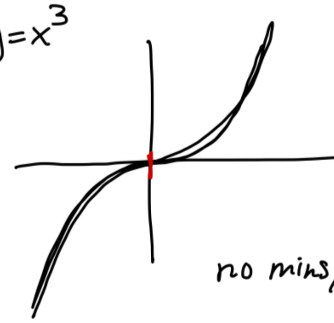
- one abs min
- no max.

$y=\frac{1}{x}$



no mins/max at all

$y=x^3$



no mins/no maxs

Don't obsess on this aspect

3. A critical ~~number~~ of $f(x)$ is an x-value, c, in the interior of the domain where $f'(x)=0$ or $f'(x)$ is undefined

Don't forget!!

4. First, find the domain and all critical numbers. Then, identify all local and/or absolute maxima and minima. Use technology to sketch the graphs and confirm your answers.

(a) $f(x) = e^x(x-2)^2$

D: $(-\infty, \infty)$

$f'(x) = e^x(x-2)^2 + e^x(2)(x-2)$

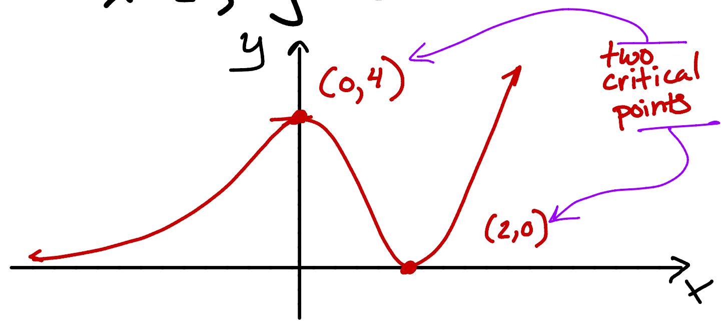
$= e^x(x-2)(x-2+2)$

$= xe^x(x-2)$.

Crit #'s $x=0, x=2$

at $x=0$, $y=e^0(-2)^2=4$ ← local max

$x=2$, $y=0$ ← abs. min



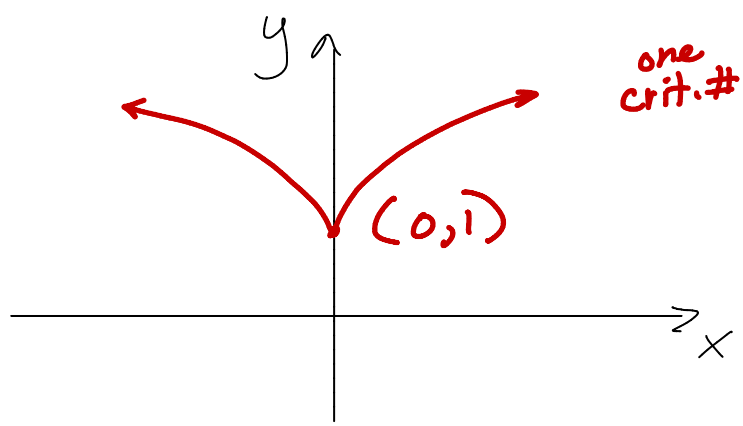
(b) $f(x) = (x-2)^{2/3} + 1$

D: $(-\infty, \infty)$

$f'(x) = \frac{2}{3}(x-2)^{-1/3} = \dfrac{2}{3\sqrt[3]{x-2}}$

crit. #: $x=2$

at $x=2$, $y=1$ ← abs min



(c) $f(x) = \frac{x^2}{(x-1)^2}$ D: $(-\infty,1)\cup(1,\infty)$

$f'(x) = \dfrac{(x-1)^2(2x) - x^2\cdot 2\cdot(x-1)}{(x-1)^4}$

$= \dfrac{(x-1)(2x) - 2x^2}{(x-1)^3}$

$= \dfrac{2x^2-2x-2x^2}{(x-1)^3} = \dfrac{-2x}{(x-1)^3}$

crit #: $x=0$ (Note $x=1$ is not in domain!)

at $x=0$, $y=0$ ← abs. min.