goals: To understand

- the difference between a secant line and a tangent line.
- how to use secant lines to estimate the slope of a tangent line.
- how to use average velocity to estimate instantaneous velocity.
- why our present tools force us to *estimate* slope or instantaneous velocity and not calculate it explicitly.

1. REVIEW: Write the equation of the line through the points $P(-3,1)$ and $Q(2,4)$.

We need a point: say $P(-3,1)$
and slope $m = \frac{\Delta y}{\Delta x} = \frac{1-4}{-3-2}$

$= \frac{-3}{-5} = \frac{3}{5}$

answer: $y-(-3) = \frac{3}{5}(x-1)$

$y+3 = \frac{3}{5}(x-1)$

$y = -3 + \frac{3}{5}(x-1)$

$y = \frac{3}{5}x - \frac{18}{5}$

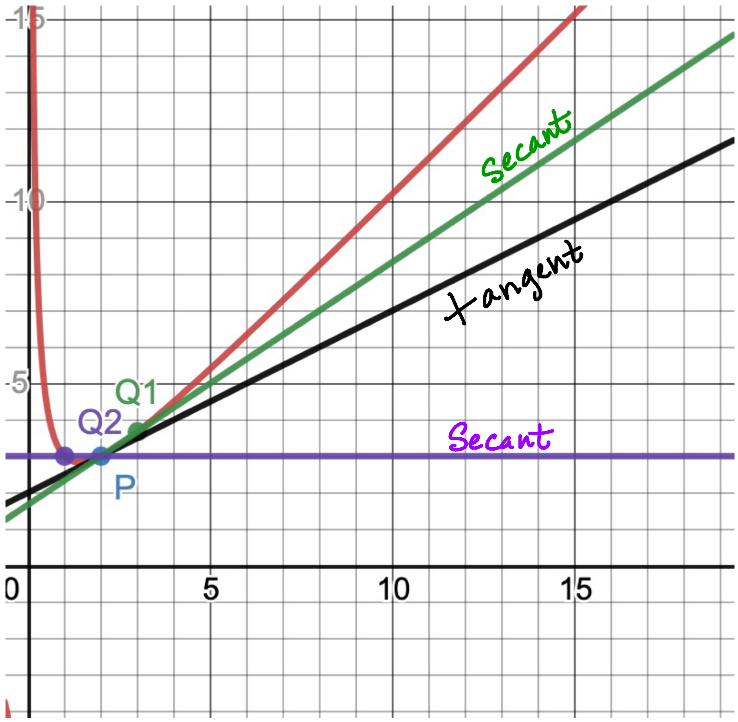
All are acceptable forms of a line.

2. The point $P(2,3)$ lies on the graph of $f(x) = x + \frac{2}{x}$.

   (a) If possible, find the slope of the secant line between the point P and each of the points with x values listed below. For each estimate, find the slope to 4 decimal places. NOTE: You do not need the graph of the function to answer this numerical question.

point Q		slope of secant line PQ
x-value	y-value	PQ
$x=4$	4.5	0.7500
$x=3$	$3.\overline{6}$	$0.\overline{6}$
$x=2.5$	3.3	0.6000
$x=2.25$	3.1388	0.5555...
$x=2.1$	3.05238	0.52380
$x=0$	undefined	~
$x=1$	3	0
$x=1.5$	$2.8\overline{3}$	$0.\overline{3}$
$x=1.75$	2.892857	0.42857
$x=1.9$	2.95263	0.47368

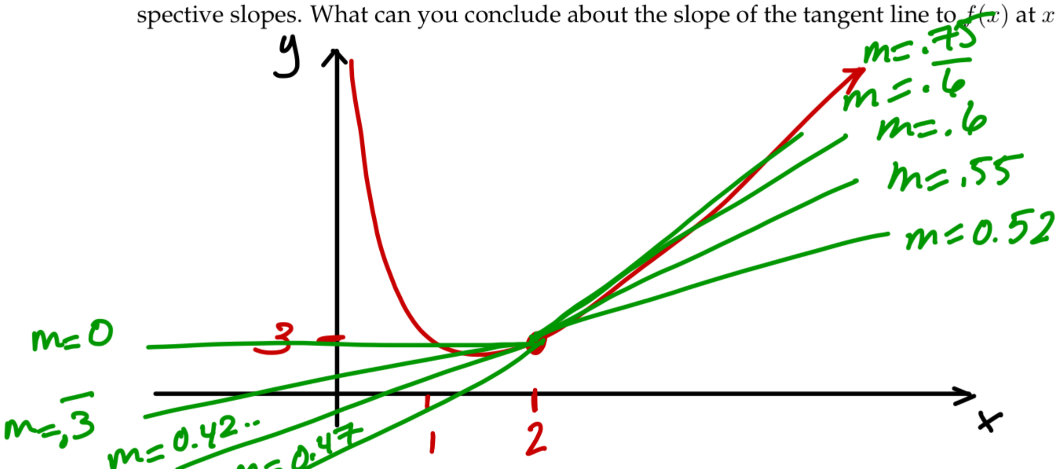
(b) Now, use technology to sketch a rough graph $f(x)$ on the interval $(0, 5]$ and add the secant lines from part a. (Your graph may be messy...It's ok.) Label the secant lines with their respective slopes. What can you conclude about the slope of the tangent line to $f(x)$ at $x = 2$?



← from Desmos

by hand by Jill

spective slopes. What can you conclude about the slope of the tangent line to $f(x)$ at $x = 2$?



(c) Write a best guess for the equation of the line tangent to $f(x)$ at point P. Is your equation plausible?

(c) Write a best guess for the equation of the line tangent to $f(x)$ at point P. Is your equation plausible?

point $(2,3)$

estimate of slope: $m = \frac{1}{2}$

line: $y - 3 = \frac{1}{2}(x-2)$

$y = 3 + \frac{1}{2}(x-2)$ or

$y = \frac{1}{2}x + 2$

3. The table shows the position of a cyclist after accelerating from rest.

t (hours)	0	0.5	1	1.5	2	2.5	3	3.5	4
d (miles)	0	6.2	13.4	23.1	33.4	44.6	54.7	62.6	70

(a) What is the cyclist's average velocity on the 4 hours of the bike ride?

$$\text{avg. vel.} = \frac{\Delta\text{position}}{\Delta\text{time}} = \frac{\Delta d}{\Delta t} = \frac{70-0}{4-0} = \frac{70}{4} = \frac{70 \text{ miles}}{4 \text{ hours}} = \boxed{17.5 \text{ miles/hour}}$$

(b) Estimate the cyclist's average velocity in miles per hour on each of the time intervals below:

i. $[0, 1.5]$

$$\text{avg. vel} = \frac{23.1-0}{1.5-0} = \frac{23.1}{1.5} = 15.4 \text{ mi/hr}$$

ii. $[0.5, 1.5]$

$$\text{avg. vel} = \frac{23.1-6.2}{1.5-0.5} = 16.9 \text{ mi/hr}$$

iii. $[1, 1.5]$

$$\text{avg. vel} = \frac{23.1-13.4}{1.5-1} = 19.4 \text{ mi/hr}$$

iv. $[1.5, 2]$

$$\text{avg. vel} = \frac{33.4-23.1}{2-1.5} = 20.6 \text{ mi/hr}$$

v. $[1.5, 2.5]$

$$\text{avg. vel} = \frac{44.6-23.1}{2.5-1.5} = 21.5 \text{ mi/hr}$$

vi. $[1.5, 3]$

$$\text{avg. vel} = \frac{54.7-23.1}{3-1.5} = 21.1 \text{ mi/hr}$$

smaller intervals of time close to $t=1.5$

smaller & smaller intervals close to 1.5

looks to approach $\approx$ 20 mi/hr

(c) The calculations above can be used to estimate the *instantaneous* velocity of the cyclist at what time? What would your estimate be?

at what time? around $t=1.5$

Estimate cyclist's instantaneous velocity at $t=1.5$ to be around $\boxed{20 \text{ mi/hr}}$

BONUS: If you understood what we did in class today, you should be able to answer the questions below.

4. In words, what is a secant line, what is a tangent line and how are they different?

A secant line is a line between two different points on a graph.

A tangent line to a graph is a line through one point on a graph that has the same slope as the slope of the graph right at that point.

Slopes of secant lines are easy to find using $\frac{\Delta y}{\Delta x}$. The slopes of tangent lines require work.

5. Justify the assertion that the problem of finding the slope of the tangent to a graph at a point is the same problem as finding the instantaneous velocity of an object given its position.

Slope is $\frac{\Delta y}{\Delta x}$ for two points $P(x,y)$ and $Q(x_1,y_1)$.

Average velocity is $\frac{\Delta \text{position}}{\Delta \text{time}}$ for two moments in time:

$A(t_1,s_1)$, $B(t_2,s_2)$.

So the calculation is the same. We just think of the variables as representing different things.

6. Explain why we can't just use our formula for slope (ie $m = \frac{\Delta y}{\Delta x}$) to find the slope of the tangent line?

We only know 1 point on the tangent line. But you need two DIFFERENT points to use $\frac{\Delta y}{\Delta x}$.