LECTURE NOTES 2-1: THE TANGENT AND VELOCITY PROBLEMS

The importance of a good question.

QUESTION 1: Given the graph of a function $y = f(x)$ and a point P on this graph, how do you *define* and *find* the equation of the tangent line to the graph at P?

QUESTION 2: Given the position of an object (say a cell phone) at any time, how do you *define* and *find* the velocity of the object at a particular instant (say the moment your child launches it off a cliff)?

Some Facts:

- These questions are old. (200BC or older depending on your interpretation)
- These questions are hard, taking more than a thousand years and untold numbers of mathematicians to answer.
- Before finding solid mathematical ground, some of its ideas were even more controversial than Donald Trump's tweets are today!
- Attempts to answer these two questions is *part* of what led to the development/discovery of Calculus.
- The ideas you learn in calculus explain planetary motion or where a projectile will land or predict how fast an infection will spread.
- **Most importantly and perhaps obviously,** *the questions that motivated the development of calculus go a long way to explaining the definitions and applications we see later*

Example 1: Let $f(x) = (10 - x^2)/2$.

(a) Sketch a LARGE graph of $f(x)$ in the space to the right. Include any x- or y-intercepts.

(b) Let P be the point on the curve where $x = 1$ and let Q be the point on the curve where $x = 3$. Find the y-coordinate for P and Q and plot them on your graph above.

(c) DEFINITION: A *secant line* on a graph is simply the line determined by two points on the graph. Find the EQUATION of the secant line determined by the points P and Q and graph it above.

(d) Label the line you just plotted above with its *slope.*

(e) For the FIVE points $Q_1,\ Q_2,\ Q_3,\ Q_4,\ Q_5$ with x-coordinates $2,\ 1.5,\ 1.25,\ 1.125, 1.0625$, find the y-coordinate, plot the point, plot the secant line determined by P and Q_i, and label the line with its slope.

(f) Sketch what YOU think the tangent to $f(x)$ at the point P should look like...???

(g) What do you observe about the relationship between the secant lines you **calculated** and the tangent line you **guessed at**?

(h) What is the significance of the **words in bold** in the previous question?

(i) What PART of the tangent line is indicated by the sequence of secant lines? _____

(j) Write the *equation* of the tangent line to $f(x)$ at P. Does this answer seem reasonable? Why or why not?

(k) In *plain old ENGLISH SENTENCES* how would you explain (step-by-step) how to find the *equation* of the tangent line?

(l) In the previous exercise, we chose points (Q_i's) on the *right* of the point P, what would happen if we had chosen points on the *left*?