LECTURE NOTES 2-2: THE LIMIT OF A FUNCTION

Things to Know:

- The intuitive definitions of a *limit* and a *one-sided limit*.

- How to find a (one-sided) limit using a calculator or the graph of the function, including infinite limits.

- How to find limits for piecewise-defined functions.

- How to distinguish between the various ways a limit may *not* exist.

- Understand how using a calculator can give an incorrect answer when evaluating a limit.

Intuitive Idea and Introductory Examples

(Note that this is motivated by our discussion of tangent lines and instantaneous velocity.)

Say: "the limit of $f(x)$, as x approaches a is L"

Write:

It means:

$\boxed{\text{EXAMPLE 1:}}$ Use calculation to guess $\displaystyle\lim_{x \to 2} \frac{x-2}{x^2-x-2}$.

What does the table above tell you about the *graph* of $y = \frac{x-2}{x^2-x-2}$?

EXAMPLE 2: [Why do all the calculation? Just pick a number really close to "a," right???!!]

$$\text{Use calculation to guess } \lim_{t \to 0} \frac{\sqrt{t^2 + 9} - 3}{t^2}.$$

Let's just pick numbers super-close to $a = 0$, say ± 0.000001 :

t	-0.000001	0	0.000001
$f(t)$		DNE	

Hint: Always be skeptical! Why can't this be right and what went wrong?

EXAMPLE 3: [Sample points may not illustrate the big picture. Theory will be useful.]

$$\text{Use calculation to guess } \lim_{\theta \to 0} \sin\left(\frac{\pi}{\theta}\right).$$

x	-0.1	-0.001	-0.0001	0	0.0001	0.001	0.01
$f(x)$							

Do you believe your answer?

Practice Problems

1. For each problem below, fill out the chart of values, then use the values to *guess* the value of the limit. Finally rate your confidence level on a 0 to 3 scale where (0 = I'm sure this is wrong) and (3 = I'm sure this is right.)

(a) $\lim\limits_{\theta \to 0} \dfrac{\sin \theta}{\theta} = \boxed{}$

confidence? _____

x					0				
$f(x)$									

(b) $\lim\limits_{x \to 2} f(x) = \boxed{}$ where $\begin{cases} |x-1| & x \le 2 \\ x+1 & x > 2 \end{cases}$

confidence? _____

x					2				
$f(x)$									

(c) $\lim\limits_{x \to 0} \dfrac{e^{2x} - 1}{x} = \boxed{}$

confidence? _____

x	-0.5	-0.1	-0.01	-0.001	-0.0001	0	0.0001	0.001	0.01	0.1	0.5
$f(x)$											

Say: "the limit as x approaches a *on the left* is L";
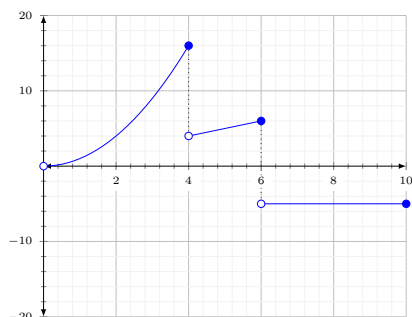
Write:

It means

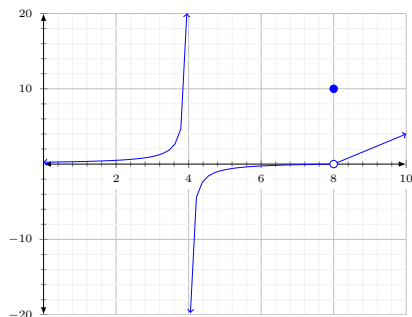Say: "the limit as x approaches a *on the right* is L";

Write:

It means

EXAMPLE 4: The function $g(x)$ is graphed below. Use the graph to fill in the blanks.

(a) $\displaystyle\lim_{x \to 4^-} f(x) =$ _____

(b) $\displaystyle\lim_{x \to 4^+} f(x) =$ _____

(c) $\displaystyle\lim_{x \to 4} f(x) =$ _____

(d) $f(4) =$ _____

(e) $\displaystyle\lim_{x \to 8} f(x) =$ _____

(f) $f(8) =$ _____

Practice Problems

2. The function $g(x)$ is graphed below. Use the graph to fill in the blanks.

(a) $\displaystyle\lim_{x \to 4^-} f(x) =$ _____

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(c) $\displaystyle\lim_{x \to 4} f(x) =$ _____

(d) $f(4) =$ _____

(e) $\displaystyle\lim_{x \to 8} f(x) =$ _____

(f) $f(8) =$ _____

Write the equation of any vertical asymptote:

EXAMPLE 5: Sketch the graph of an function that satisfies *all* of the given conditions. Could there be more than one correct answer?

$$\lim_{x \to 0^-} f(x) = 1 \qquad \lim_{x \to 0^+} f(x) = -2 \qquad \lim_{x \to 4^-} f(x) = 3$$

$$\lim_{x \to 4^+} f(x) = 0 \qquad f(0) = -2 \qquad \qquad f(4) = 1$$

EXAMPLE 6: Determine the limit. Explain your answer.

1. $\lim\limits_{x \to 5^+} \dfrac{2 + x}{x - 5}$

2. $\lim\limits_{x \to 5^+} \dfrac{2 + x}{5 - x}$

3. $\lim\limits_{x \to (\pi/2)^+} \dfrac{\sec x}{x}$