LECTURE NOTES 2-3: CALCULATING LIMITS USING THE LIMIT LAWS

REVIEW:

1. What does it mean to write $\lim_{x \to a} f(x) = L$?

2. Given $f(x)$ and a how did we find the limit L or show that it doesn't exist?

GOALS:

- Learn a whole bunch of *general principles* about calculating limits.
- Correct and careful application of these principles will allow us to (a) avoid the tedious calculation from Section 2-2 and (b) avoid the mistakes and pitfalls of relying on numerical approximations.

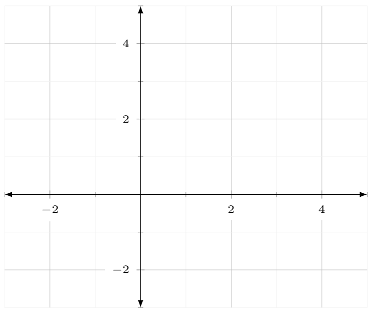
Limit Laws (Table 1)

In the rules below c is a constant and $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a}(x)$ both exist.

formal notation	in English sentences
1. $\lim\limits_{x \to a}[f(x) + g(x)] =$	1.
2. $\lim\limits_{x \to a}[f(x) - g(x)] =$	2.
3. $\lim\limits_{x \to a}[cf(x)] =$	3.
4. $\lim\limits_{x \to a}[f(x) \cdot g(x)] =$	4.
5. $\lim\limits_{x \to a}[f(x)/g(x)] =$	5.

EXAMPLE 1:

1. Graph $f(x) = x$ and $g(x) = 3.8$ on the axes below:



2. Use the graphs to evaluate the limits below:

$\lim_{x\to 2} f(x) =$ ______

$\lim_{x\to 2} g(x) =$ ______

3. Do the limits $\lim_{x\to 2} f(x)$ and $\lim_{x\to 2} g(x)$ exist? Why?

4. Use the limit laws and part 2. above to evaluate $\lim_{x\to 2}[8x - 3.8]$. Justify your steps.

5. Use the limit laws and part 2. above to evaluate $\lim_{x\to 2} x^5$. Justify your steps.

Limit Laws (Table 2)

In the rules below c is a constant, n is a positive integer, and $\lim_{x\to a} f(x)$ exists.

1. $\lim_{x\to a}(f(x))^n =$	2. $\lim_{x\to a} c =$
3. $\lim_{x\to a} x =$	4. $\lim_{x\to a} x^n =$
5. $\lim_{x\to a} \sqrt[n]{x} =$	6. $\lim_{x\to a} \sqrt[n]{f(x)} =$

EXAMPLE 2: Evaluate $\lim_{x \to -3} \frac{\sqrt{x^2 - 5}}{4 - 2x}$ and justify your steps.

EXAMPLE 3: Let $f(x) = \frac{x^2+1}{2x-4}$.

a. Find $\lim_{x \to -1} f(x)$.

b. Find $f(-1)$

c. Find $\lim_{x \to 2^+} f(x)$.

d. Find $f(2)$

e. **T or F**: $\lim_{x \to -1} f(x) = f(-1)$.

f. **T or F**: $\lim_{x \to 2} f(x) = f(2)$.

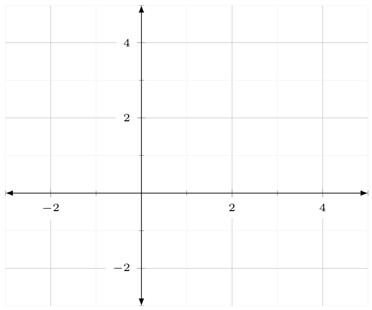
g. Fill in the blank in the statement of the DIRECT SUBSTITUTION PROPERTY:

If $f(x)$ is a polynomial or rational function and a is ____________________________, then

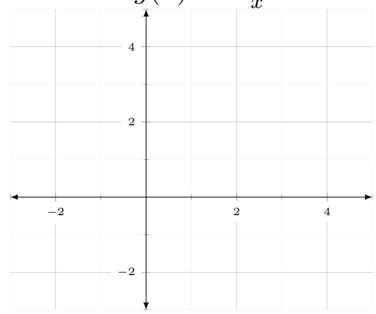
$$\lim_{x \to a} f(x) =$$

EXAMPLE 4:

a. Sketch $f(x) = x + 1$



b. Sketch $g(x) = \frac{x^2+x}{x}$.



c. Find $f(0)$.

d. Find $g(0)$.

e. Find $\lim\limits_{x\to 0} f(x)$.

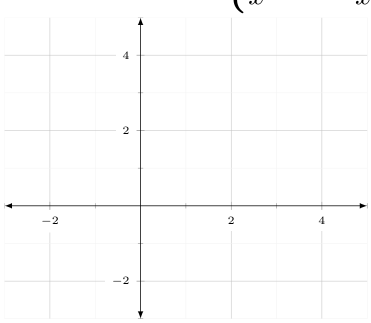
f. Find $\lim\limits_{x\to 0} g(x)$.

g. For what x-values is $f(x) = g(x)$? For what x-values is $f(x) \neq g(x)$?

h. Explain how $\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x)$ even though $f(0) \neq g(0)$.

EXAMPLE 5: Sketch the graph of each function below and find the indicated limits, if they exists. If the limits do not exist, explain why they do not exist.

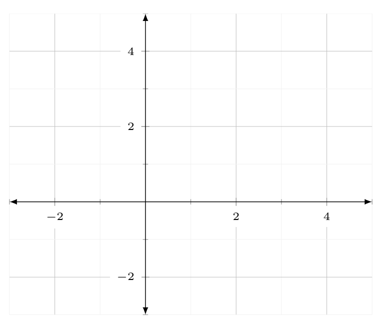
a. Sketch $f(x) = \begin{cases} e^x - 1 & x < 0 \\ 2 & x = 0 \\ x^2 & x > 0 \end{cases}$



$\lim\limits_{x\to -2} f(x)$.

$\lim\limits_{x\to 0} f(x)$.

b. $g(x) = \frac{|x|}{x}$.



$\lim\limits_{x\to 3} g(x)$.

$\lim\limits_{x\to 0} g(x)$.

In general, describe the relationship between the TWO-sided limit and each of the ONE-sided limits.

QUESTION: What do we do with limits that result in $0/0$?

EXAMPLE 6:

(a) $\displaystyle\lim_{x\to -4} \frac{x^2+5x+4}{x^2+3x-4}$

(b) $\displaystyle\lim_{x\to 16} \frac{4-\sqrt{x}}{16x-x^2}$

(c) $\displaystyle\lim_{h\to 0} \frac{\frac{1}{3+h}-\frac{1}{3}}{h}$