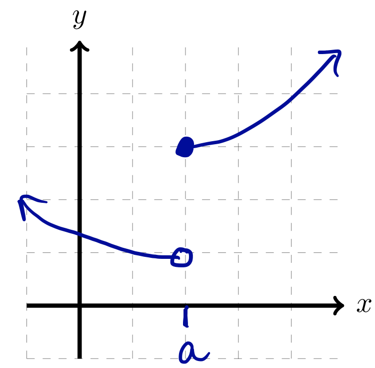
LECTURE NOTES 2-5: CONTINUITY (DAY 1)

QUESTION: In plain old words in English, what *should* it mean to say

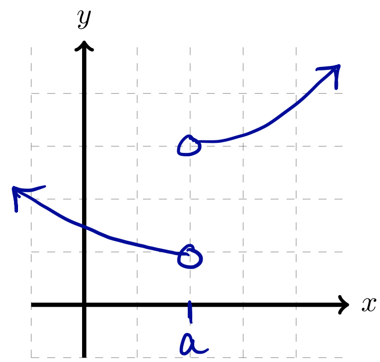
the function $f(x)$ is continuous at $x = a$?

around $x=a$, the graph of $f(x)$ should have no holes or jumps.

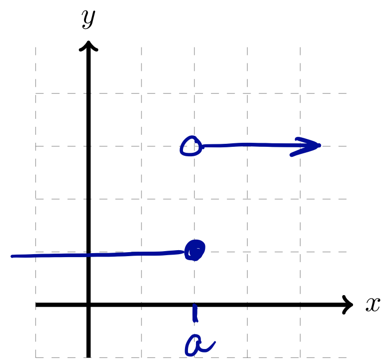
On the axes below, draw some pictures of graphs that are NOT continuous at some point and label that point with the x-value a. Succinctly describe why it's not continuous.



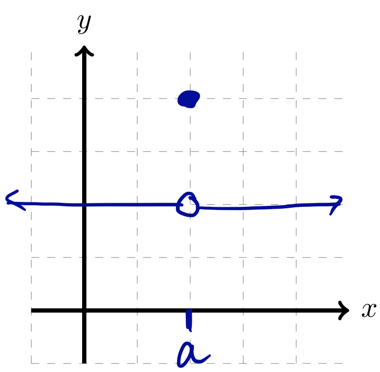
jump dis continuity



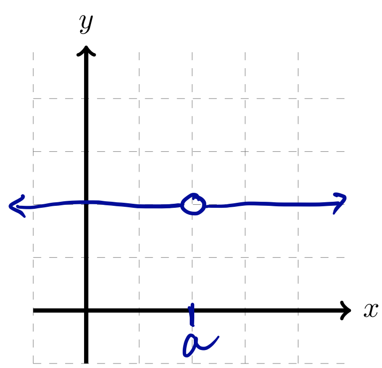
jump discontinuity



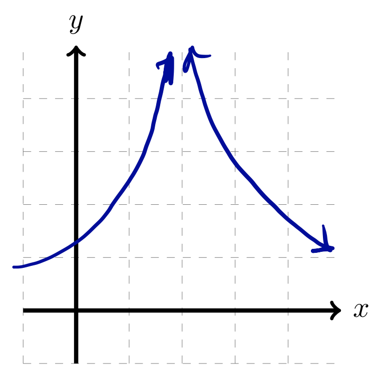
jump discontinuity



jump discontinuity



removable dis continuity



Infinite discontinuity

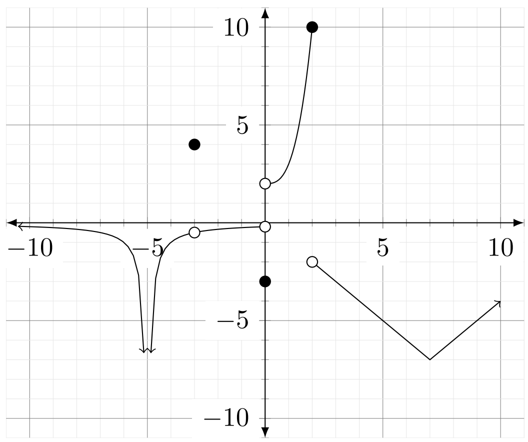
DEFINITION: A function $f(x)$ is continuous at the number $x = a$ if

$$\boxed{\lim_{x \to a} f(x) = f(a)}$$

(i.e. what happens "close" to $x=a$ better be the same as what happens at $x=a$.)

Practice Problems: For each function below, state the numbers for which $f(x)$ is continuous and the numbers for which $f(x)$ is discontinuous. For each point of discontinuity, explain why it is discontinuous.

1. $f(x)$ is graphed below. Assume arrows indicate the function continues in that general direction.



(a) continuous?

$(-\infty,-5)\cup(-5,-3)\cup(-3,0)\cup(0,2)\cup(2,\infty)$

(b) discontinuous and why?

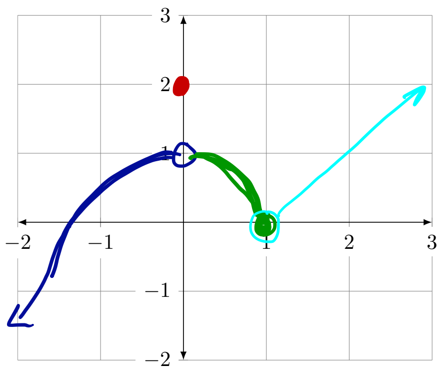
$x=-5$ (infinite) $\lim_{x\to-5} f(x) = DNE$

$x=-3$ (jump) $\lim_{x\to-3} f(x) = \frac{1}{2} \neq 4 = f(-3)$

$x=0$ (jump) $\lim_{x\to 0} f(x) = DNE$

$x=2$ (jump) $\lim_{x\to 2} f(x) = DNE$

2. $g(x) = \begin{cases} \cos x & x < 0 \\ 2 & x = 0 \\ 1-x^2 & 0 < x \leq 1 \\ x-1 & 1 < x \end{cases}$



(a) continuous?

$(-\infty,0)\cup(0,\infty)$

(b) discontinuous and why?

$x=0$.

$\lim_{x\to 0} g(x) = 1 \neq 2 = g(0)$.

a jump discontinuity

3. $h(x) = \frac{x^3-8}{x^2-4} = \frac{(x-2)(x^2+2x+4)}{(x+2)(x-2)}$

(a) continuous?

$(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

(b) discontinuous and why?

$x=-2$, infinite discontinuity, $\lim_{x\to-2} h(x)$ does not exist.

$x=2$, removable discontinuity, $f(2)$ does not exist.