LECTURE NOTES 2-5: CONTINUITY (PART 2)

REVIEW: A function $f(x)$ is continuous at the number $x = a$ if

Sketch a function with domain $(\infty, -1) \cup (-1, \infty)$ that has a removable discontinuity at $x = -1$, an infinite discontinuity at $x = 0$, and a jump discontinuity $x = 1$.

GOALS: In this lesson, we will practice using the definition of continuity, define right- and left-continuity, and learn (& apply) several very powerful theorems concerning continuous functions.

DEFINITION: A function $f(x)$ is **continuous from the right** at the number $x = a$ if

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A function $f(x)$ is **continuous from the left** at the number $x = a$ if

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QUESTION: Look at your picture above and determine all the a-values for which your function is continuous from the right and those for which your function is continuous from the left.

QUESTION: Why would we want one-sided continuity?

QUESTION: Assume $f(x)$ and $g(x)$ are BOTH continuous at $x = a$, what do you think should be true about the new function $H(x) = f(x) + g(x)$ and how would you JUSTIFY your intuition?

THEOREMS 4, 5, AND 7 (as numbered in your textbook) all tell us that a large family of familiar functions are continuous. Below we will list this collection. The numbering aligns with the textbook theorem.

4.1

5a

4.2

4.3

5b

4.4

4.5

7

EXAMPLES:

1. Determine the intervals over which the function $f(x) = \frac{3e^x + \tan x}{5x}$ is continuous and *justify* your answer using the Theorems above.

2. Evaluate $\lim_{x \to \pi/4} \dfrac{3e^x + \tan x}{5x}$ and *justify* your strategy.

THEOREMS 8 AND 9 (as numbered in your textbook) tell us that continuity is preserved by function *composition* **provided** the resulting function is defined.

EXAMPLE: Determine all x-values for which the function $f(x) = \ln\left(\frac{1}{x} - 1\right)$ is continuous.

EXAMPLES:

1. Determine the domain of the function $g(r) = \tan^{-1}(1 + e^{-r^2})$ and explain why $g(r)$ is continuous at every number in its domain.

2. Use continuity to evaluate $\lim\limits_{x \to 4} 3^{\sqrt{x^2-2x-4}}$.

3. Let $f(x) = 1/x$ and $g(x) = 1/x^2$. (a) Find $(f \circ g)(x)$. (b) Explain why $f \circ g$ is not continuous everywhere.

THE INTERMEDIATE VALUE THEOREM: Suppose $f(x)$ is a function such that

- $f(x)$ is continuous on $[a, b]$,
- $f(a) \neq f(b)$, and
- N is a number between $f(a)$ and $f(b)$,

then,

EXAMPLES:

1. Use the Intermediate Value Theorem to show that the equation $x^4 + x - 3 = 0$ must have a root in the interval $(1, 2)$.

2. Give an example of a function $f(x)$ that is defined for every number in the interval $[0, 2]$ such that $f(0) = 0$, $f(2) = 1$ but there does not exist a single x-value in the interval $(0, 2)$ such that $f(x) = 1/2$.