LECTURE NOTES 2-5: CONTINUITY (PART 2)

REVIEW: A function f(x) is continuous at the number x = a if



Sketch a function with domain $(\infty, -1) \cup (-1, \infty)$ that has a removable discontinuity at x = -1, an infinite discontinuity at x = 0, and a jump discontinuity x = 1.

GOALS: In this lesson, we will practice using the definition of continuity, define right- and left-continuity, and learn (& apply) several very powerful theorems concerning continuous functions.

DEFINITION: A function f(x) is continuous from the right at the number x = a if

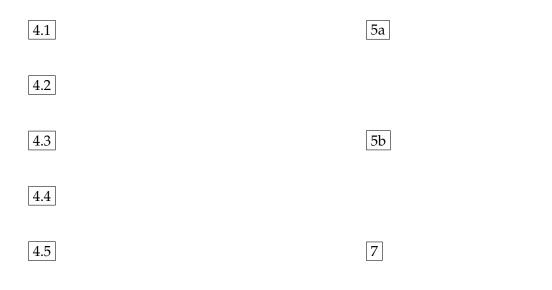
A function f(x) is **continuous from the left** at the number x = a if



QUESTION: Look at your picture above and determine all the *a*-values for which your function is continuous from the right and those for which your function is continuous from the left.

QUESTION: Assume f(x) and g(x) are BOTH continuous at x = a, what do you think should be true about the new function H(x) = f(x) + g(x) and how would you JUSTIFY your intuition?

THEOREMS 4, 5, AND 7 (as numbered in your textbook) all tell us that a large family of familiar functions are continuous. Below we will list this collection. The numbering aligns with the textbook theorem.



EXAMPLES:

1. Determine the intervals over which the function $f(x) = \frac{3e^x + \tan x}{5x}$ is continuous and *justify* your answer using the Theorems above.

2. Evaluate $\lim_{x \to \pi/4} \frac{3e^x + \tan x}{5x}$ and *justify* your strategy.

THEOREMS 8 AND 9 (as numbered in your textbook) tell us that continuity is preserved by function *composition* **provided** the resulting function is defined.

EXAMPLE: Determine all *x*-values for which the function $f(x) = \ln(\frac{1}{x} - 1)$ is continuous.

EXAMPLES:

1. Determine the domain of the function $g(r) = \tan^{-1}(1 + e^{-r^2})$ and explain why g(r) is continuous at every number in its domain.

2. Use continuity to evaluate $\lim_{x \to 4} 3^{\sqrt{x^2 - 2x - 4}}$.

3. Let f(x) = 1/x and $g(x) = 1/x^2$. (a) Find $(f \circ g)(x)$. (b) Explain why $f \circ g$ is not continuous everywhere.

THE INTERMEDIATE VALUE THEOREM: Suppose f(x) is a function such that

- f(x) is continuous on [a, b],
- $f(a) \neq f(b)$, and
- *N* is a number between f(a) and f(b),

then,

EXAMPLES:

1. Use the Intermediate Value Theorem to show that the equation $x^4 + x - 3 = 0$ must have a root in the interval (1, 2).

2. Give an example of a function f(x) that is defined for every number in the interval [0,2] such that f(0) = 0, f(2) = 1 but there does not exist a single *x*-value in the interval (0,2) such that f(x) = 1/2.