LECTURE: 2-6 LIMITS AT INFINITY [PART 1]

Intuitive Definition of a Limit at Infinity Let f be a function defined on some interval (a, ∞) or $(-\infty, a)$. Then

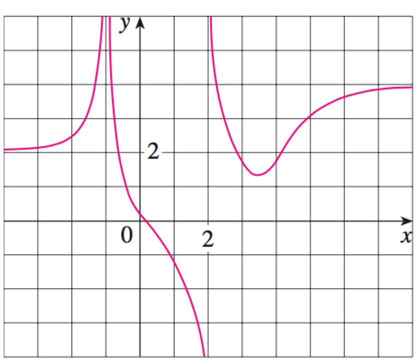
means that the values of $f(x)$ can be made arbitrarily close to L by requiring x to be big enough...

The line $y = L$ is called a **horizontal asymptote** of the curve $y = f(x)$ if either

$$\lim_{x \to \infty} f(x) = L \quad \text{or} \quad \lim_{x \to -\infty} f(x) = L$$

Example 1: Sketch a graph of $y = \tan^{-1} x$ and find the $\lim_{x \to \infty} \tan^{-1} x$ and $\lim_{x \to -\infty} \tan^{-1} x$.

Example 2: Find the infinite limits, limits at infinity, and asymptotes for the function f whose graph is shown below.



Example 2: Find the following limits.

a) $\lim_{x \to \infty} \frac{1}{7x+1}$

b) $\lim_{x \to \infty} \sin x$

c) $\lim_{x \to \infty} 3e^{-x}$

Big idea: Often, terms in function we are interested in finding the limit of will reduce to "$\frac{k}{0}$" or "$\frac{k}{\infty}$" when we attempt to evaluate them. We can think of these terms as going to...

How to Determine Limits at Infinity: Divide the numerator and denominator by the highest common power between the numerator and denominator.

Example 3: Find the limit.

(a) $\lim_{x \to \infty} \frac{2x+5}{x-4}$

(b) $\lim_{x \to \infty} \frac{x+4}{x^2+x-3}$

Example 4: Evaluate the following limits.

(a) $\lim_{x\to\infty} \frac{2x^2+5}{3x^2+1}$

(b) $\lim_{x\to\infty} \frac{2x+5}{3x^2+1}$

(c) $\lim_{x\to\infty} \frac{2x^3+5}{3x^2+1}$

Example 5: Find the following limits at infinity.

(a) $\lim_{x\to\infty} \frac{1+5e^x}{7-e^x}$

(b) $\lim_{x\to\infty} [\ln(2+x) - \ln(1+x)]$

Example 6: Find the limit.

(a) $\lim_{x\to\infty} \frac{x+2}{\sqrt{9x^2+1}}$

(b) $\lim_{x\to\infty} \frac{\sqrt{3x^6-x}}{x^3+1}$