LECTURE: 2-8 THE DERIVATIVE AS A FUNCTION

The function

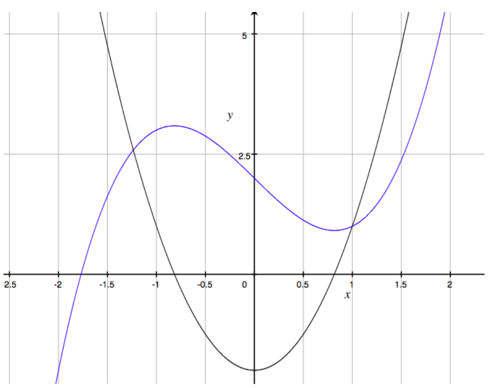
$$f'(x) = \underline{\hspace{4cm}}$$

is called the **derivative of** f. The value of f' at x can be interpreted geometrically as the $\underline{\hspace{2.5cm}}$ of the tangent line to f at the point $(x, f(x))$. *Note: f' is called the derivative because it has been derived from f using the limit operation defined above. The domain of f' is the set of all x such that this limit exists and may be smaller than the domain of f.*

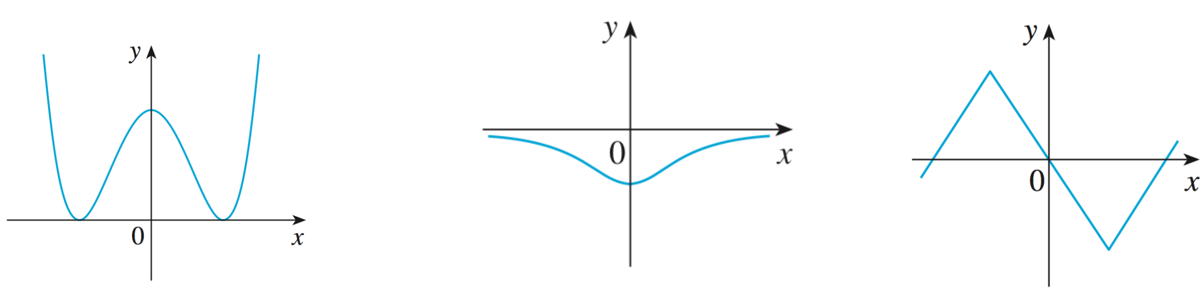
Example 1: Let $f(x) = x^3 - 2x + 2$.

(a) Find a formula for $f'(x)$.

(b) Illustrate this formula by comparing the graphs of $f(x)$ and $f'(x)$, which are shown below.



Example 2: The graph of f is given below. Use it to sketch the graph of the derivative f'.



Example 3: If $f(x) = \sqrt{x-5}$ find the derivative of f. State the domain of f and f'.

Example 4: If $f(x) = \dfrac{2-x}{5+2x}$ find $f'(x)$. State the domain of f and f'.

Other Notations for $f'(x)$

A function f is **differentiable at** a if $f'(a)$ exists. It is **differentiable on an open interval** (a,b) [or $(a,\infty), (-\infty,a)$ or $(-\infty,\infty)$] if it is differentiable at every number in the interval.

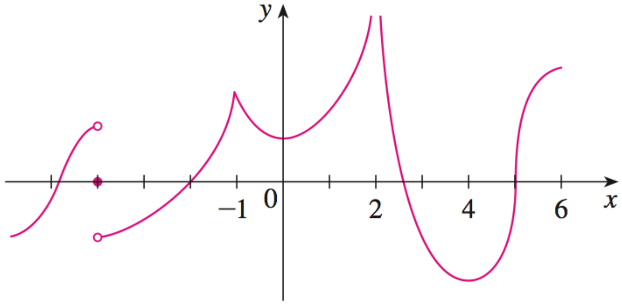
Example 5: Where do the following functions fail to be differentiable?

(a) $f(x) = |x|$

(b) $f(x) = \dfrac{1}{x}$

Example 6: Where does $f(x) = \sqrt[3]{x}$ fail to be differentiable? Graph $f(x)$ and explain what the behavior of the tangent line is near this point.

Example 7: A graph of a function $f(x)$ is shown below. State, with reasons, where the function f is not differentiable.



Differentiable Implies Continuous: If f is differentiable at a, then f is continuous at a.

Proof:

Is the converse of this theorem true? That is, if f is continuous at $x = a$ does this imply that f is differentiable at a? Why or why not?

Higher Derivatives

If f is a differentiable function then its derivative f' is a function, so f' may also have a derivative of its own, denoted by $(f')' = f''$, called the second derivative. Similarly you can also take the derivative of the second derivative, called the third derivative f'''.

Example 8: Given $f(x) = x^3 - 2x + 2$, find and interpret $f''(x)$, $f'''(x)$ and $f^{(4)}(x)$. *(Note: We found $f'(x) = 3x^2 - 2$ in an earlier example.)*