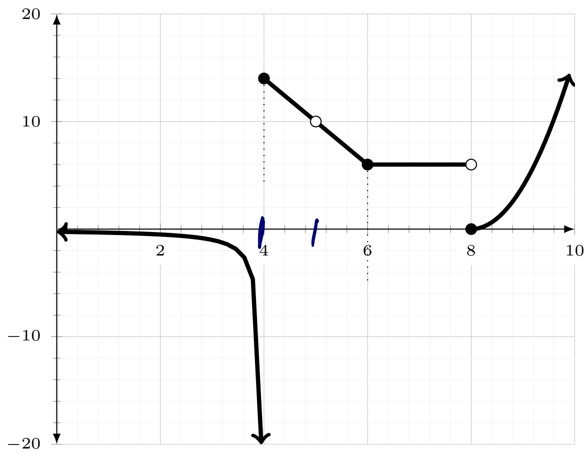
Lecture Notes: Chapters 1 & 2 Review

Practice Problems:

1. Use the graph of $f(x)$ below to answer the following questions.



(a) Assuming the arrows on the graph indicate a continued curve in that direction, make an educated guess at the domain of the function $f(x)$.

$(-\infty,4)\cup(4,5)\cup(5,\infty)$

(b) Find all x-values in the domain of $f(x)$ for which $f(x)$

i. fails to be continuous.

just $x=8$ (4 & 5 are not in domain)

ii. fails to be differentiable.

$x=8$, $x=6$

not cts   corner

(c) Evaluate the following limits or explain why they do not exist.

(i) $\lim_{x\to4^-} f(x) =$ $-\infty$

(ii) $\lim_{x\to4^+} f(x) =$ 14

(iii) $\lim_{x\to4} f(x) =$ DNE

(iv) $\lim_{x\to5} f(x) =$ 10

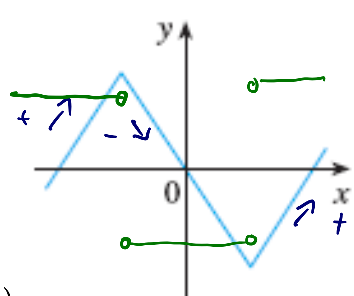
(v) $\lim_{x\to6} f(x) =$ 6

(vi) $\lim_{x\to7} f(x) =$ 6

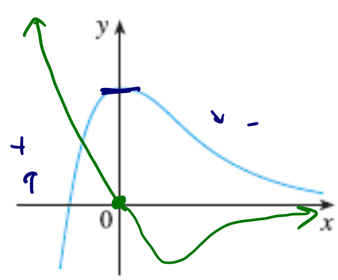
(vi) $\lim_{x\to8} f(x) =$ DNE

(vii) $\lim_{x\to8^-} f(x) =$ 6

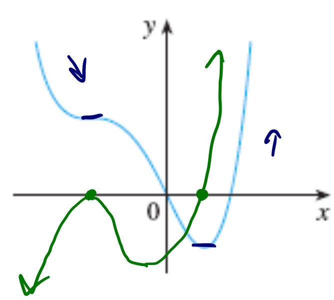
2. Given the functions $f(x)$ shown below, graph each derivative $f'(x)$. [Superimposed is good.]



(a)



(b)



(c)

3. Evaluate the following limits. Show your work. *Make sure you are writing your mathematics correctly and clearly.*

(a) $\lim\limits_{t\to 2}\left(\dfrac{t^2-4}{t^3-3t+5}\right)^3 = \left(\lim\limits_{t\to 2}\dfrac{t^2-4}{t^3-3t+5}\right)^3$

cts at $t=2$

$= \left(\dfrac{4-4}{8-6+5}\right)^3 = \boxed{0}$

(b) $\lim\limits_{x\to 4^-}\dfrac{x^2+3x}{x^2-x-12} = \lim\limits_{x\to 4^-}\dfrac{x(\cancel{x+3})}{(x-4)(\cancel{x+3})}$

$= \lim\limits_{x\to 4^-}\dfrac{x}{x-4} \boxed{= -\infty}$

Numer approaches +4
Denom. $\to 0$, but negative

(c) $\lim\limits_{x\to -3}\dfrac{x^2-4x}{x^2-x-12} = \lim\limits_{x\to 3}\dfrac{x(\cancel{x-4})}{(\cancel{x-4})(x+3)}$

$= \lim\limits_{x\to -3}\dfrac{x}{x+3} = \boxed{DNE}$

the limits from the left and right are different.
(∞ & $-\infty$ respectively)

(d) $\lim\limits_{h\to 0}\dfrac{(h-5)^2-25}{h} = \lim\limits_{h\to 0}\dfrac{h^2-10h+\cancel{25}-\cancel{25}}{h}$

$= \lim\limits_{h\to 0} h-10 = \boxed{-10}$

4. For each function below, determine all the values in the domain of the function for which the function is continuous.

(a) $f(x) = \begin{cases} \frac{3}{x+5} & x < 1 \\ \frac{x+1}{2} & 1 \le x \le 3 \\ x^2-7 & 3 < x \end{cases}$

$x=3$: $f(3) = \frac{3+1}{2} = 2$ ✓

$\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} x^2-7 = 2$ ✓

cts at $x=3$

$x=1$: $f(1) = \frac{1+1}{2} = 1$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-}\frac{3}{x+5} = \frac{3}{6} = \frac{1}{2}$

$\lim\limits_{x\to 1} f(x) \neq f(1)$ disc. at $x=1$

Disc. at $x=-5$ ($\div$ by 0)

$\boxed{(-\infty,-5)\cup(-5,1)\cup(1,\infty)}$

(b) $g(x) = \frac{2^x+1}{\sqrt{1-x}}$ cts on its domain.

$1-x>0$ $x<1$ ✓

cts on $\boxed{(-\infty, 1)}$

5. Find the limit or show that it does not exist.

(a) $\lim\limits_{x\to-\infty} \frac{2-x}{3x^2-x} = \lim\limits_{x\to\infty} \frac{2+x}{3x^2+x} \frac{\cdot 1/x}{\cdot 1/x}$

$$= \lim_{x\to\infty} \frac{\cancel{2/x}^{\to 0}+1}{3x+\cancel{1/x}^{\to 0}} = \boxed{0}$$

$-$ "$\frac{1}{\infty}$"

(b) $\lim\limits_{x\to\infty}[\ln(1+x^2)-\ln(1+x)] = \lim\limits_{x\to\infty} \ln\left(\frac{1+x^2}{1+x}\right)$

$$= \ln\left(\lim_{x\to\infty}\left(\frac{1+x^2}{1+x} \frac{\cdot 1/x}{\cdot 1/x}\right)\right)$$

$$= \ln\left(\lim_{x\to\infty} \frac{\cancel{1/x}^{0}+x}{\cancel{1/x}^{0}+1}\right) = \boxed{\infty}$$

$(\lim\limits_{x\to\infty} \ln x = \infty)$

(c) $\lim\limits_{x\to\infty} \frac{3x^2+2x}{\sqrt{x^4+2x}} \frac{\cdot 1/x^2}{\cdot 1/x^2}$

$\underline{x^2 = \sqrt{x^4}}$

$$= \lim_{x\to\infty} \frac{3+2/x}{\sqrt{1+2/x^3}}$$

$$= \frac{3+0}{\sqrt{1+0}} = \boxed{3}$$

6. The displacement (in feet) of a particle moving in a straight line is given by $s(t) = \dfrac{2}{t} + 10$ where t is measured in seconds.

(a) Find the average velocity from $t = 1$ to $t = 4$ and include units with your answer.

$$V_{ave} = \frac{s(4) - s(1)}{4-1}$$

$$= \frac{\frac{2}{4} + 10 - \left(\frac{2}{1} + 10\right)}{3}$$

$$= \frac{\frac{1}{2} + \cancel{10} - 2 - \cancel{10}}{3}$$

$$= \frac{-\frac{3}{2}}{3} = \boxed{-\tfrac{1}{2}}\ \text{ft/sec}$$

"3 halves divided by 3"

(b) Find the instantaneous velocity of the particle when $t = 2$ and include units with your answer.

$$v(t) = s'(t) = \lim_{h \to 0} \frac{\frac{2}{t+h} + 10 - \left(\frac{2}{t} + 10\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{t}{t} \cdot \frac{2}{t+h} - \frac{2}{t} \cdot \frac{(t+h)}{(t+h)}}{h}$$

$$= \lim_{h \to 0} \left(\frac{2t - 2(t+h)}{t(t+h)}\right) \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-2\cancel{h}}{t(t+h)\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-2}{t(t+h)} = \frac{-2}{t^2}$$

$$v(2) = \frac{-2}{2^2} = \boxed{\frac{-1}{2}\ \text{ft/sec}}$$