

A car is traveling down the road, and we have measured the distance it traveled at various times over a period of one hour. In this worksheet we will discuss how to convert distance measurements into speed measurements for the car.

To begin, you will need to access a Google Docs spreadsheet that contains the car measurements. To do this:

1. Go to Google Drive associated with UAF:  
docs.alaska.edu.
2. You will see a new item shared with you titled "Calc I Velocity". Open it up.
3. You won't be able to edit this spreadsheet. Instead, click File, and select Make a Copy. Selecting "OK" will open a now editable spreadsheet.

Now answer the following questions.

1. How far did the car travel during the hour?

$$\sim 36.45 \text{ mi}$$

2. If you had to give just one answer, how fast would you say the car was traveling during the hour? Express your answer first in units of miles per hour (because that will feel most comfortable), but then also in miles per minute (because these units will be used for the rest of the worksheet).

$$36.45 \text{ mi/hr! (or } 0.6075 \text{ mi/min)}$$

3. The speed you just computed is called the *average speed* over the hour. Now compute the average speed over the first half hour and the average speed over the second half hour. Express your answer in units of miles per minute. You can use the spreadsheet to do these computations for you, if you would like. If you have questions on how to do this, ask!

$$\frac{12.99 \text{ mi}}{30 \text{ min}} \approx 0.433 \text{ mi/min} \quad \frac{36.45 - 12.99 \text{ mi}}{30 \text{ min}} \approx 0.782 \text{ mi/min}$$

4. Your computations should show that the average speeds are not the same during the two half-hour periods. Which one was slower? How could you have predicted this from looking at the graph of distance traveled?

2nd half faster. From 30-60 min, graph appears "steeper"

5. Just looking at the graph, decide if the car is traveling faster around the 20 minute mark, or around the 40 minute mark.

$$40 \text{ minute! "steeper"}$$

6. Now compute the average speeds (in miles per minute) over the time interval from 20 to 25 minutes, and the time interval from 40 to 45 minutes.

$$\frac{11.49 - 11.05}{5} = 0.086 \text{ mi/min} \quad \frac{26.76 - 21.24}{5} \approx 1.105 \text{ mi/min}$$

7. Although the spreadsheet only shows, right now, the distance traveled at 5 minute intervals, you can determine the distance at other times, too. For example, to compute the distance traveled after 3 minutes and 45 seconds, go to a blank cell and enter =DIST(3.75). After a pause, you should see that the distance traveled at that point is about 3.67 miles. Now use your newly-found power to compute the distance traveled after 41 minutes and after 42 minutes, and the average speed over the time interval from 41 to 42 minutes.

At 41 minutes:  $\sim 22.34$  miles

$$\frac{\text{DIST}(42) - \text{DIST}(41)}{42 - 41} \approx 1.1134 \text{ mi/min}$$

8. Let's use the notation  $d(t)$  for the distance traveled, so  $d(40) \approx 21.24$ , for example. We want to write down a formula for the average speed over the time interval  $[t_0, t_1]$ .

- (a) Using only the symbols  $d$ ,  $t_0$ ,  $t_1$  and arithmetic operations, write an expression for the total distance traveled over the time interval  $[t_0, t_1]$ .
- (b) What is the length of the time interval  $[t_0, t_1]$ ?
- (c) Write down a formula for the average speed over the time interval  $[t_0, t_1]$ .

(a)  $d(t_1) - d(t_0)$

(b)  $t_1 - t_0$

(c) 
$$\frac{d(t_1) - d(t_0)}{t_1 - t_0}$$

9. Sometimes it will be handy to use time intervals of the form  $[t_0, t_0 + h]$ , so  $t_0$  is the start of the interval and  $h$  is the length of the interval. Substitute  $t_1 = t_0 + h$  into your expression for average speed in your previous problem and simplify.

$$\frac{d(t_0 + h) - d(t_0)}{t_0 + h - t_0} = \frac{d(t_0 + h) - d(t_0)}{h}$$

10. Now let's contemplate the speed right at time 41 minutes. This corresponds to a time interval with  $t_0 = 41$  and  $t_1 = 41$  (or alternatively with  $t_0 = 41$  and  $h = 0$ ). Explain why your formulas for average speed from the previous two problems are not immediately helpful now.

If  $h = 0$ , we are dividing by 0!

11. Instead, we can work with average speeds over short time intervals near time  $t = 41$  minutes. Use the spreadsheet to compute the average speeds over the time intervals  $[41, 41 + h]$  for:

(a)  $h = 1$  minutes

1.113414664

(b)  $h = 0.1$  minutes

1.108981788

(c)  $h = 0.01$  minutes

1.108373364

(d)  $h = 0.001$  minutes

1.108310877

(e)  $h = 0.0001$  minutes

1.108304612

(f)  $h = 0.00001$  minutes

1.108303986

To answer this, you might find it helpful to make a little table in the spreadsheet with two columns, one with  $h$  and one with average speed for that value of  $h$ .

12. Write down an estimate for the for the speed of the car at time  $t = 41$  minutes to as many digits that you feel confident in, given your answers to the previous problem.

1.10830 mi/min