LECTURE: 3-10 LINEAR APPROXIMATION AND DIFFERENTIALS

Linear Approximation

Example 1: Find the linearization of $f(x) = \sqrt{x+3}$ at $a = 1$ and use it to approximate the numbers $\sqrt{3.98}$ and $\sqrt{4.05}$. Are these approximations overestimates or underestimates?

Example 2: Physicists commonly use linear approximations to simplify a non-linear function. Find the linear approximation of $f(x) = \sin x$.

Example 3: Use a linear approximation to estimate $\sqrt{99.8}$ in two different ways. Does your estimate differ?

(a) $f(x) = \sqrt{x}$, $a = 100$

(b) $f(x) = \sqrt{x + 100}$, $a = 0$

Example 4: Find the linear approximation of $f(x) = e^x \cos x$ at $a = 0$.

Differentials

Example 5: Compare the values of Δy and dy if $y = f(x) = x^3 + x^2 - 2x + 1$ and x changes

(a) from 2 to 2.10.

(b) from 2 to 2.01.

(c) What happens to Δy and dy as Δx decreases?

Example 6: Find the differential of the function.

(a) $y = \cos \pi x$

(b) $y = \dfrac{1}{(1 + 2r)^4}$

Example 7: Find the differential dy and evaluate dy for the given values of x and dx.

$y = x^3 - 6x^2 + 5x - 7$, $x = -2$, $dx = 0.1$

Example 8: The radius of a sphere was measured and found to be 21 cm with a possible error of at most 0.05 cm. What is the maximum error in using this value of the radius to compute the volume of the sphere?

Example 9: The radius of a circular disk is given at 24 cm with a maximum error in measurement of at most 0.2 cm.

(a) Use differentials to estimate the maximum error in the calculated area of the disc. Does this error seem large?

(b) What is the relative error? What is the percent error? Does the error (still) seem large?