LECTURE: 3-2 THE PRODUCT AND QUOTIENT RULES [PART 2] AND INTRO TO 3-3

Recall the derivative rules we have so far:

- **Power rule:** $(x^n)' = nx^{n-1}$
- **Constant multiple rule:** $(cf(x))' = cf'(x)$
- **Sum/difference rule:** $(f \pm g)' = f' \pm g'$
- **Product rule:** $(fg)' = fg' + f'g$
- **Quotient rule:** $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

1. Differentiate the following.

(a) $f(x) = (x - 4\sqrt{x})e^x$ $= (x - 4x^{1/2})e^x$

$$f'(x) = (1 - 2x^{-1/2})e^x + (x - 4x^{1/2})e^x$$

$$\boxed{= \left(1 - \frac{2}{\sqrt{x}} + x - 4\sqrt{x}\right)e^x}$$

(b) $y = \dfrac{\sqrt{x}}{1 + 2x}$

$$y' = \frac{(1+2x)\cdot\frac{1}{2\sqrt{x}} - 2\sqrt{x}}{(1+2x)^2}$$

$$= \frac{\frac{1}{2\sqrt{x}} + \sqrt{x} - 2\sqrt{x}}{(1+2x)^2} = \frac{\frac{1}{2\sqrt{x}} - \sqrt{x}}{(1+2x)^2} = \boxed{\frac{1 - 2x}{\sqrt{x}(1+2x)^2}}$$

(c) $g(x) = \dfrac{ax + b}{cx + d}$

$$g'(x) = \frac{(cx+d)(a) - (ax+b)(c)}{(cx+d)^2}$$

$$= \frac{\cancel{acx} + ad - \cancel{acx} - bc}{(cx+d)^2} = \boxed{\frac{ad - bc}{(cx+d)^2}}$$

2. Find the derivative in two ways: (i) product rule and (ii) first multiply out.

$f(x) = (x + x^2)(x^{-1} + 3)$

i) $(x+x^2)(-x^{-2}) + (1+2x)(x^{-1}+3)$

$= -x^{-1} - 1 + x^{-1} + 3 + 2 + 6x$

$= \boxed{6x + 4}$

ii) $x \cdot x^{-1} + 3x + x^2 \cdot x^{-1} + 3x^2$

$= 3x^2 + 4x + 1$

$\boxed{f'(x) = 6x + 4}$

3. Find an equation of the tangent line and normal line to the given curve $y = 2\sqrt{x}e^x + 1$ at the point $(0, 1)$.

$y' = 2\left(\sqrt{x}e^x + \frac{1}{2\sqrt{x}}e^x\right)$ $x = 0$ uh-oh... $\div$ by 0!

tangent line has undefined slope through point $(0,1)$:

i.e., $\boxed{x = 0}$ — equation of tangent line

normal line: $\boxed{y = 1}$

4. A manufacturer produces socks. The quantity q of these socks (measured in pairs of socks) that are sold are a function of the selling price p (in dollars), so we can write $q = f(p)$. Then the total revenue earned with a selling price p is $R(p) = pf(p)$.

(a) What does it mean to say $f(10) = 20,000$ and $f'(10) = 3,500$?

- $f(10) = 20,000$ means if socks are \$10 each, 20,000 pairs will be sold
- $f'(10) = 3,500$: At the price of \$10, the number of socks sold is increasing at 3500 socks/\$.

(b) Assuming the values in part (a), find $R'(10)$ and interpret your answer.

$R'(p) = (pf(p))'$ product rule!

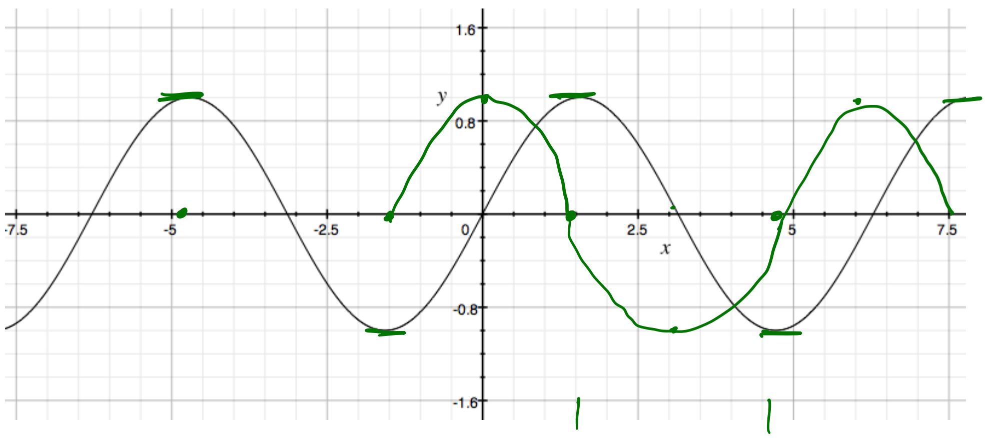
$= pf'(p) + 1 \cdot f(p)$

$R'(10) = 10(3500) + 20,000 = \boxed{55,000}$

When the price is \$10, revenue is increasing at a rate of \$55,000 per \$1 increase in price

3-3: Intro to Derivatives of Trigonometric Functions

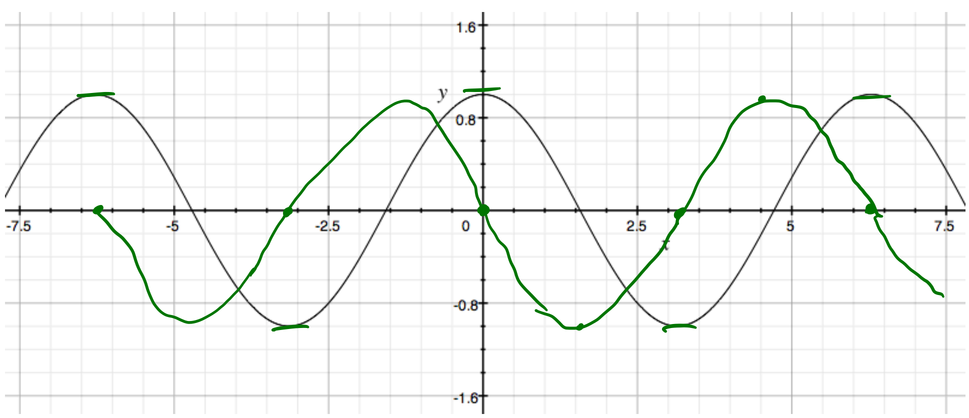
Example 1: Use the graph of $y = \sin x$ to sketch a graph of y'. Guess what y' is.



$y = \sin x$

$y' = \cos x$!!

Example 2: Use the graph of $y = \cos x$ to sketch a graph of y'. Guess what y' is.



$y' = -\sin x$!!

Example 3: Using the derivatives we just found, let us find the derivative of $f(x) = \tan x$. What is the domain of $f'(x)$?

$$f(x) = \frac{\sin x}{\cos x}$$ $\leftarrow$ quotient rule!

$$f'(x) = \frac{\cos x(\cos x) - \sin x(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$$