LECTURE: 3-3 DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

Recall last time we found $\frac{d}{dx}(\sin x) = \cos x$ and $\frac{d}{dx}(\cos x)$.

Example 1: Using the derivative of $\sin x$ and $\cos x$ find derivatives of:

(a) $y = \cot x$

(b) $y = \csc x$

Derivatives of Trigonometric Functions:

- $\frac{d}{dx}(\sin x) =$ ____________
- $\frac{d}{dx}(\cos x) =$ ____________
- $\frac{d}{dx}(\tan x) =$ ____________
- $\frac{d}{dx}(\csc x) =$ ____________
- $\frac{d}{dx}(\sec x) =$ ____________
- $\frac{d}{dx}(\cot x) =$ ____________

Example 2: Find the second derivatives of the following functions:

(a) $g(t) = 4\sec t + \tan t$.

(b) $y = x^2 \sin x$.

Example 3: Find an equation of the tangent line to the curve $y = \dfrac{1}{\sin x + \cos x}$ at the point $(0, 1)$.

Example 4: For what values of x does the graph of $f(x) = x + 2\sin x$ have a horizontal tangent?

Example 5: Differentiate $f(x) = \dfrac{\sec x}{1 - \tan x}$ and determine where the tangent line is horizontal.

Generalized Product Rule: How does the product rule genearlize to more than two functions? For example, what is the derivative of $y = f(x)g(x)h(x)$?

Example 6: Differentiate $h(\theta) = \theta^2 \tan\theta \sec\theta$.

Example 7: Find the 51st derivative of $f(x) = \sin x$. Specifically, find the first four or five derivatives and look for a pattern.

Example 8: A mass on a spring vibrates horizontally on a smooth level surface. Its equation of motion is $x(t) = 8\sin t$, where t is in seconds and x is in centimeters.

(a) Find the velocity at time t.

(b) Find the position and velocity of the mass at time $t = 2\pi/3$. In what direction is it moving at this time?

Example 9: A ladder 12 feet long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \frac{\pi}{6}$.