

LECTURE: 3-4 THE CHAIN RULE

When you have a function that is a composite function, like $y = \sqrt{x^2 + 1}$, the formulas we have so far do not let us find y' . However, if you write your composite function as $f \circ g$, we have a formula for the derivative.

The Chain Rule: If f and g are differentiable and $F = f \circ g$, then F is differentiable and

$$F'(x) = f'(g(x))g'(x).$$

Example 1: Write the composite function in the form $f(g(x))$ and then find y' .

(a) $y = (1 + 3x)^9$

(b) $y = \frac{1}{(x^2 + 2x - 5)^9}$

Example 2: Write the composite function in the form $f(g(x))$. Then, find y' .

(a) $y = \cos(x^3)$

(b) $y = \cos^3(x)$

Example 3: Find the derivative of $f(x) = (2x - 1)^6(x^3 - 2x + 1)^3$

Example 4: Find the derivative of $f(x) = \left(\frac{x+5}{2x-1}\right)^5$.

Example 5: Find the derivative of the following functions.

(a) $y = e^{\sec x}$

(b) $y = \sin(\sin(\sin x))$

Example 6: Let $F(x) = f(g(x))$, where $f(-2) = 8$, $f'(-2) = 4$, $f'(5) = 3$, $g(5) = -2$, and $g'(5) = 6$, find $F'(5)$.

Example 7: Find the derivative of the following functions.

(a) $g(x) = \sqrt[5]{x^3 - 1}$

(b) $h(x) = \sin^5(4x^2)$

Formula: Derivative of $y = b^x$:

$$\frac{d}{dx}(b^x) =$$

Example 8: Find the derivative of the following functions.

(a) $y = 5^x$

(b) $f(x) = 10^{\cos x}$

(c) $g(x) = e^{-2x^2}$

Example 9: Find the derivative of the following functions.

(a) $f(x) = 5^{3x^2}$

(b) $y = \sin \sqrt{3x}$

Example 10: Find the points on the graph of the function $f(x) = 2 \cos x + \cos^2 x$ at which the tangent is horizontal.

Example 11: Find the 100th derivative of $y = \sin(5x)$.

Example 12: A model for the length of day (in hours) in Philadelphia on the t -th day of the year is

$$L(t) = 12 + 2.8 \sin \left[\frac{2\pi}{365}(t - 80) \right].$$

Use this model to compare the number of hours of daylight is increasing in Philadelphia on January 15th ($t = 15$) and March 21st ($t = 80$).

Example 13: Use the product rule and chain rule to prove the quotient rule.

Example 14: Find the derivatives of the following functions.

(a) $y = \cos^2(\cot(2x))$

(b) $y = x^3 e^{-1/x^2}$

Example 15: Find an equation of the tangent line to the curve $y = 3^{\sin x}$ at the point where $x = 0$.