LECTURE: 3-4 THE CHAIN RULE

When you have a function that is a composite function, like $y = \sqrt{x^2 + 1}$, the formulas we have so far do not let us find y'. However, if you write your composite function as $f \circ g$, we have a formula for the derivative.

The Chain Rule: If f and g are differentiable and $F = f \circ g$, then F is differentiable and

$$F'(x) = f'(g(x))g'(x).$$

Example 1: Write the composite function in the form f(g(x)) and then find y'.

(a)
$$y = (1+3x)^9$$

(b)
$$y = \frac{1}{(x^2 + 2x - 5)^9}$$

Example 2: Write the composite function in the form f(g(x)). Then, find y'.

(a)
$$y = \cos(x^3)$$

(b)
$$y = \cos^3(x)$$

Example 3: Find the derivative of $f(x) = (2x - 1)^6(x^3 - 2x + 1)^3$

Example 4: Find the derivative of $f(x) = \left(\frac{x+5}{2x-1}\right)^5$.

Example 5: Find the derivative of the following functions.

(a)
$$y = e^{\sec x}$$

(b)
$$y = \sin(\sin(\sin x))$$

Example 6: Let F(x) = f(g(x)), where f(-2) = 8, f'(-2) = 4, f'(5) = 3, g(5) = -2, and g'(5) = 6, find F'(5).

Example 7: Find the derivative of the following functions.

(a)
$$g(x) = \sqrt[5]{x^3 - 1}$$

(b)
$$h(x) = \sin^5(4x^2)$$

Formula: Derivative of $y = b^x$:

$$\frac{d}{dx}(b^x) =$$

Example 8: Find the derivative of the following functions.

(a)
$$y = 5^x$$

(b)
$$f(x) = 10^{\cos x}$$

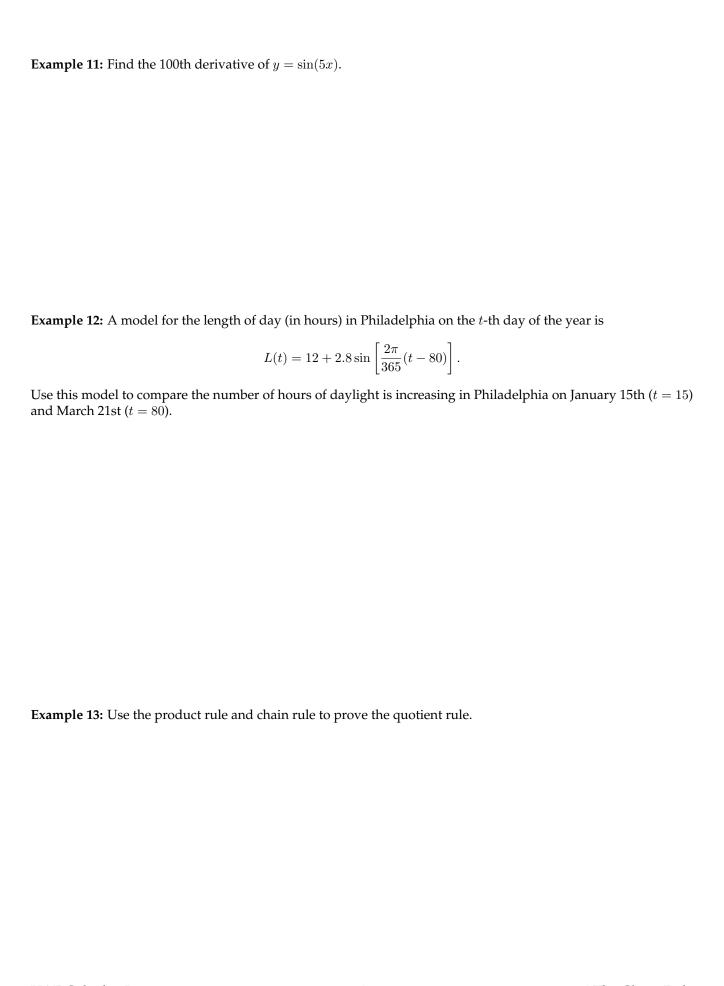
(c)
$$g(x) = e^{-2x^2}$$

Example 9: Find the derivative of the following functions.

(a)
$$f(x) = 5^{3^{x^2}}$$

(b)
$$y = \sin \sqrt{3x}$$

Example 10: Find the points on the graph of the function $f(x) = 2\cos x + \cos^2 x$ at which the tangent is horizontal.



UAF Calculus I 4 3-4 The Chain Rule

Example 14:	Find the	derivatives	of the	following	functions

(a)
$$y = \cos^2(\cot(2x))$$

(b)
$$y = x^3 e^{-1/x^2}$$

Example 15: Find an equation of the tangent line to the curve $y = 3^{\sin x}$ at the point where x = 0.