LECTURE: 3-6 DERIVATIVES OF LOGARITHMIC FUNCTIONS [PART 2]

Note the difference between derivatives of powers of x and exponentials (where x shows up in the *exponent*).

Derivative Rules: Let n and a be constants. (Note, there is no rule when there is a variable in the base *and* the exponent.)

- $\frac{d}{dx}x^n = nx^{n-1}$
- $\frac{d}{dx}a^x = (\ln a)a^x$

When you have a variable in both the base and the exponent you **must** use logarithmic differentiation to find the derivative of the function.

Example 1: Find the derivatives of the following functions using logarithmic differentiation.

(a) $y = x^{2/x}$

$$\ln y = \ln x^{2/x}$$

derivative!

$$\ln y = \frac{2}{x}\cdot \ln x$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{2}{x}\cdot\frac{1}{x} - \frac{2}{x^2}\ln x$$

$$\frac{dy}{dx} = \left(\frac{2}{x^2} - \frac{2}{x^2}\ln x\right)\cdot y$$

$$\boxed{\frac{dy}{dx} = \left(\frac{2}{x^2} - \frac{2}{x^2}\ln x\right)\cdot x^{2/x}}$$

(b) $y = (\ln x)^{\cos x}$

$$\ln y = \cos x \cdot \ln(\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = -\sin x\cdot \ln(\ln x) + \cos x\left(\frac{1}{\ln x}\cdot\frac{1}{x}\right)$$

$$\boxed{\frac{dy}{dx} = \left(-\sin x \ln(\ln x) + \frac{\cos x}{x\ln x}\right)(\ln x)^{\cos x}}$$

Example 2: Find an equation of the tangent line to $f(x) = \ln(x + \ln x)$ at $x = 1$.

$$f'(x) = \frac{1}{x + \ln x}\left(1 + \frac{1}{x}\right)$$

$$y = \ln(1 + \ln 1) = \ln(1+0) = 0$$

$$m = \frac{1}{1 + \ln 1}\left(1 + \frac{1}{1}\right) = \frac{1}{1}(2) = 2 \qquad (1, 0)$$

$$y - 0 = 2(x-1)$$

$$\boxed{y = 2x - 2}$$

Example 3: Let $f(x) = cx + \ln(\sin x)$. For what value of c is $\underline{f'(\pi/4) = 6}$?

$$f'(x) = c + \frac{1}{\sin x} \cdot (\cos x)$$

$$\Rightarrow c + \frac{\cos \pi/4}{\sin \pi/4} = 6$$

$$c + \frac{\sqrt{2}/2}{\sqrt{2}/2} = 6 \qquad c + 1 = 6 \qquad \boxed{c = 5}$$

$$\boxed{f(x) = 5x + \ln(\sin x)}$$

3-7 Rates of Change in the Natural and Social Sciences [a start]

Physics example: A particle moves according to the law of motion $s = f(t) = t^4 - 4t + 1$, where t is measured in seconds and s is measured in meters.

(a) Find the velocity at time t. What is the velocity after 2 seconds?

$$v(t) = f'(t) = 4t^3 - 4$$

$$f'(2) = 4(2)^3 - 4 = \boxed{28 \text{ m/s}}$$

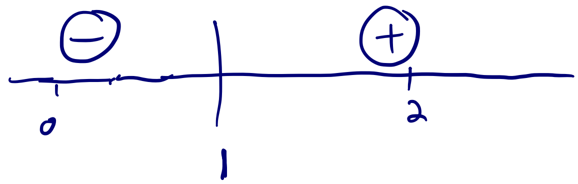
(b) When is the particle at rest? - velocity is 0!

$$4t^3 - 4 = 0 \qquad t^3 = 1 \Rightarrow \boxed{t = 1 \text{ sec}}$$

$$4t^3 = 4$$

(c) When is the particle moving forward (in the positive direction)?

$v(t) > 0$ - test values! Sign chart



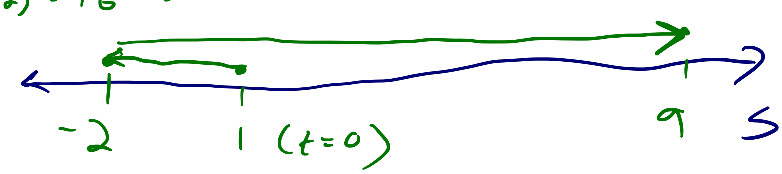
on $[1, \infty)$, $v(t) > 0$, so the position is increasing (i.e., moving forward)

(d) Draw a diagram to illustrate the motion of the particle.

① Plot start $t = 0$, $s(0) = 1$

② Plot turning point(s) $t = 1$, $s(1) = 1 - 4 + 1 = -2$

③ Final direction $t = 2$, $s(2) = 16 - 8 + 1 = 9$



(e) Find the total distance traveled in the first 4 seconds.

0-1 seconds: 3 meters

$t = 4$, position is $s(4) = 4^4 - 4(4) + 1$ 241

so 1-4 seconds, 2 + 241 meters

Total: $\boxed{246 \text{ meters}}$