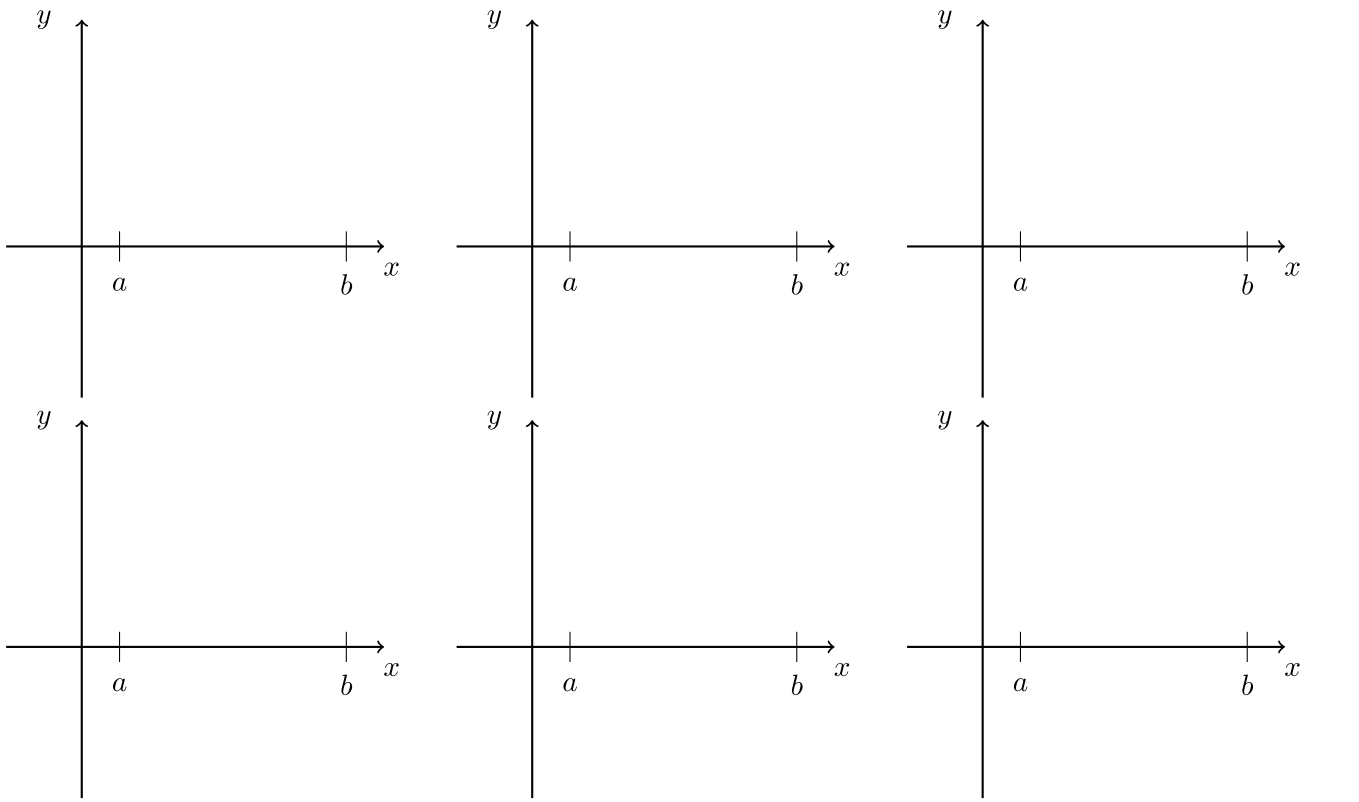
LECTURE NOTES: 4-2 THE MEAN VALUE THEOREM (PART 1)

MOTIVATING EXAMPLES: Draw several examples of graphs of functions such that (i) the domain is $[a, b]$ and (ii) $f(a) = f(b)$. Note you are not *required* to make sketches that are continuous or differentiable, though you may choose to do so.



QUESTION 1: What is the difference between a *conjecture* and a *Theorem* in a mathematics course?

QUESTION 2: State in plain old English (or draw a picture) to explain what it means for the graph of $f(x)$ if you know $f'(c) = 0$.

QUESTION 3: Based on our examples on the previous page and your knowledge of graphs more broadly, what requirements would be needed to *guarantee* the existence of an x-value c in the open interval (a, b) such that $f'(c) = 0$?

ROLLE'S THEOREM: If [blank box],

then there is a number c in the interval (a, b) such that $f'(c) = 0$.

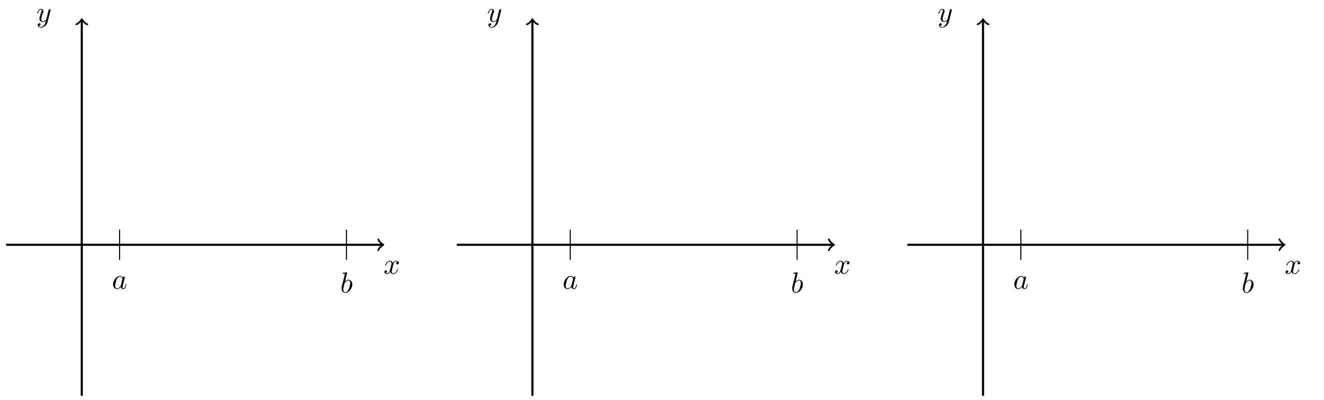
QUESTION 4: Now that we see a pattern, can we give an argument for why that pattern should hold? (HINT: What does the Extreme Value Theorem say again??)

EXAMPLE 1: Consider $f(x) = x^3 - 2x^2 - 4x + 2$ on the interval $[-2, 2]$.

1. Verify that the function $f(x)$ satisfies the hypothesis of Rolle's Theorem on the given interval.

2. Find all numbers c that satisfy the conclusion of Rolle's Theorem.

MOTIVATING EXAMPLES: Draw several examples of graphs of functions such that (i) the domain is $[a, b]$, (ii) $f(x)$ is continuous on $[a, b]$, **and** (iii) $f(x)$ is differentiable on $[a, b]$. We are *not* assuming that $f(a) = f(b)$.



QUESTION 5: In each picture above, draw (or in some other way identify) the quantity:

$$\frac{f(b) - f(a)}{b - a}.$$

What would this quantity be if Rolle's Theorem applied?

THE MEAN VALUE THEOREM: If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there is a number c in the interval (a, b) such that

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OBSERVATION: The Mean Value Theorem is just Rolle's Theorem if you turn your head sideways.

QUESTION 6: Assume that $f(x)$ is continuous and differentiable on the interval $[a, b]$ and assume there exists some x-value d in (a, b) such that $f(d) > f(a)$, can you draw any conclusion about $f'(x)$? Why or why not?

THEOREM 5: If $f'(x) = 0$ for all x in the interval (a, b), then

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QUESTION 7: How would you explain why this theorem is true? (Hint: See your answer to Question 6!)

QUESTION 8: If $f(x)$ gives the *position* of an object as a function of time, what "common sense" idea is the MVT telling us? Theorem 5?