Lecture Notes: 4-2 The Mean Value Theorem (part 2)

Warm-up Problems:

1. State Rolle's Theorem and draw a picture illustrating it.

2. State the Mean Value Theorem and draw a picture illustrating it.

3. "Rolle's Theorem? We don't need no stinking Rolle's Theorem."

4. Consider $f(x) = 1/x$ on the interval $[1, 3]$.

    (a) Verify that the function $f(x)$ satisfies the hypothesis of the Mean Value Theorem on the given interval.

    (b) Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

    (c) Sketch the graph to show that your answer above are correct.

5. Construct an example of a specific function $f(x)$ and interval $[a, b]$ such that there are exactly three numbers c in (a, b) satisfying the Mean Value Theorem.

6. Fill in the blank below and draw a picture illustrating this theorem.

    If $f'(x) = 0$ for all x in the interval (a, b), then ______________________________.

ONE LAST BIG IDEA:

1. Give the formulas for two *different* functions $f(x)$ and $g(x)$ such that $f'(x) = g'(x)$ and sketch these two functions on the same set of axes.

2. **Corollary 7:** If $f'(x) = g'(x)$ for all x in the interval (a, b), then

or, said another way,

PRACTICE PROBLEMS:

1. Suppose f is continuous on $[2, 5]$ and $1 \le f'(x) \le 4$ for all x in $(2, 5)$. Show that $3 \le f(5) - f(2) \le 12$.

2. For each function below, show that there is no value of c on $[0, 2]$ such that $f'(c) = \dfrac{f(2) - f(0)}{2 - 0}$. Why does this not contradict Rolle's Theorem?

   a) $f(x) = |x - 1|$

   b) $f(x) = \dfrac{1}{(x-1)^2}$

3. Two stationary patrol cars equipped with radar are 5 miles apart on a highway. As a truck passes the first patrol car, its speed is clocked at 55 miles per hour. Four minutes later, when the truck passes the second patrol car, its speed is clocked at 50 miles per hour. Prove that the truck must have exceeded the speed limit of 55 miles per hour at some time during the four minutes.