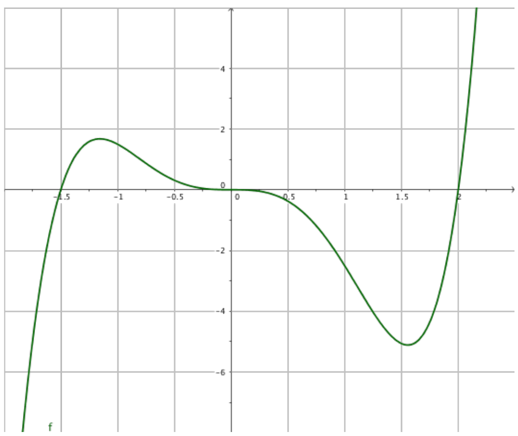
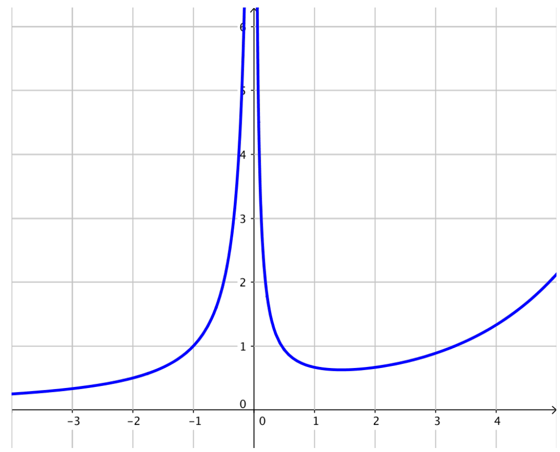
LECTURE NOTES: 4-3 HOW DERIVATIVES AFFECT THE SHAPE OF A GRAPH (PART 2)

DEFINITION: A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward or vice versa at P.

QUESTION 1: Do the following (familiar) graphs have any inflection points?





CONCAVITY TEST & INFLECTION POINTS: Let $f(x)$ be a function defined on an interval I.

a) If ____________________ (that is: ___________________________) for all x in I, then the graph of f is concave upward on I.

b) If ____________________ (that is: ___________________________) for all x in I, then the graph of f is concave downward on I.

PRACTICE PROBLEM 1: Let $f(x) = 2x^3 - 3x^2 - 12x$. Find the intervals of concavity and the inflection points.

QUESTION 2: Homer Simpson once read from the newspaper: "Here's good news! According to this eye-catching article, SAT scores are declining at a slower rate." What does this have to do with first and second derivatives?

The Second Derivative Test: Suppose f'' is continuous near c.

a) If ____________________ and ____________________, then f has a **local minimum** at c.

b) If ____________________ and ____________________, then f has a **local maximum** at c.

c) If ____________________ and ____________________, then the test is inconclusive.

PRACTICE PROBLEM 2: Let $f(x) = x^4 - 4x^3$. Find critical points, intervals of concavity, inflection points, and local maxima and minima. Use that information to sketch the curve.

LECTURE NOTES: 4-4 INDETERMINATE FORMS AND L'HOSPITAL'S RULE (PART 1)

MOTIVATING EXAMPLES: Evaluate the Chapter 2 limits below, justifying each step:

a) $\lim_{x \to 2} \frac{x^2 - 4}{x^2 - 5x + 6}$

b) $\lim_{x \to 0} \frac{\sin x}{x}$

L'Hospital's Rule If a limit has the form __________ or __________,

then

$$\lim_{x \to a} \frac{f(x)}{g(x)} =$$

provided

QUESTION 1: Determine whether or not l'Hospital's Rule applies to the MOTIVATING EXAMPLES (copied below) and if it does, apply it. Do you get the same answer?

a) $\lim_{x \to 2} \frac{x^2 - 4}{x^2 - 5x + 6}$

b) $\lim_{x \to 0} \frac{\sin x}{x}$

QUESTION 2: Why does l'Hospital's Rule work?

PRACTICE PROBLEMS: Evaluate the following limits.

1. $\displaystyle\lim_{x \to 0} \frac{\tan(5x)}{\sin(3x)}$

2. $\displaystyle\lim_{u \to \infty} \frac{e^{u/10}}{u^2}$

3. $\displaystyle\lim_{x \to 0} \frac{\cos(4x)}{e^{2x}}$

4. $\displaystyle\lim_{x \to 0} \frac{xe^x}{2^x - 1}$

5. $\displaystyle\lim_{x \to 1^+} \big(\ln(x^4 - 1) - \ln(x^9 - 1)\big)$