Lecture Notes: 4-5 Curve Sketching (Part 2)

Warm up Problem Find your copy of the Graphing Guidelines!

Practice Problems

1. Sketch the curve $y = x - 2\sin x$ on $[-2\pi, 2\pi]$.

   (a) Find the domain. $\mathbb{R}$

   (b) Find the x and y-intercepts.

   when $x=0$, $y=0$.

   when $y=0, \ldots$ solve $2\sin x = x$? hard. let it go.

   Hey! once we are done we see there sure are other solutions. We'll find them in §4.8

   (c) Find the symmetries/ periodicity of the curve.

   x, $\sin x$ both odd.

   So I expect the function to be odd.

   (d) Determine the asymptotes. none. $\lim_{x\to\infty} x - 2\sin x = \infty$, $\lim_{x\to-\infty} x - \sin x = -\infty$.

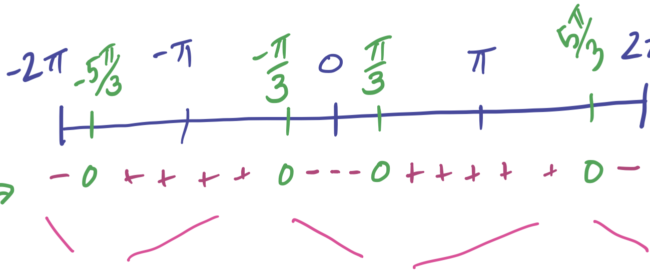
   (e,f) Determine where the function is increasing/ decreasing and find the local maximum/ minimum values

$y' = 1 - 2\cos x = 0$

$\cos x = \frac{1}{2}$ $\quad x = \frac{\pi}{3}, -\frac{\pi}{3}$

critical points in $[-2\pi, 2\pi]$ are:

$x = -\frac{5\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$



sign of y'

ANS:

y is increasing on $\left(-\frac{5\pi}{3}, -\frac{\pi}{3}\right) \cup \left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$

and decreasing on $\left(-2\pi, -\frac{5\pi}{3}\right) \cup \left(-\frac{\pi}{3}, \frac{\pi}{3}\right) \cup \left(\frac{5\pi}{3}, 2\pi\right)$.

local minimums at $x = -\frac{5\pi}{3}$, min value $-\frac{5\pi}{3} + \sqrt{3}$

at $x = \frac{\pi}{3}$, min value $\frac{\pi}{3} - \sqrt{3}$

at $x = 2\pi$, min value 2π

local maximums at $x = -2\pi$, max value -2π

at $x = -\frac{\pi}{3}$, max value $-\frac{\pi}{3} + \sqrt{3}$

at $x = \frac{5\pi}{3}$, max value $\frac{5\pi}{3} - \sqrt{3}$

(g) Find the intervals of concavity/inflection points.

$y' = 1 - 2\cos x$

So $y'' = 2\sin x$.

So $y'' = 0$ in $[-2\pi, 2\pi]$

when $x = -2\pi, -\pi, 0, \pi, 2\pi$

$-2\pi \quad -\pi \quad 0 \quad \pi \quad 2\pi$

$0 \; + \; 0 \; - \; 0 \; + \; 0 \; - \; 0$ ← sign of y''

answer:

y is concave up on $(-2\pi, \pi) \cup (0, \pi)$ and concave down on $(-\pi, 0) \cup (\pi, 2\pi)$.

inflection points:

x	$-\pi$	0	π
y	$-\pi$	0	π

(h) Sketch the curve.

points to plot:

$(-2\pi, -2\pi)$

$\left(-\frac{5\pi}{3}, \approx -3.5\right)$

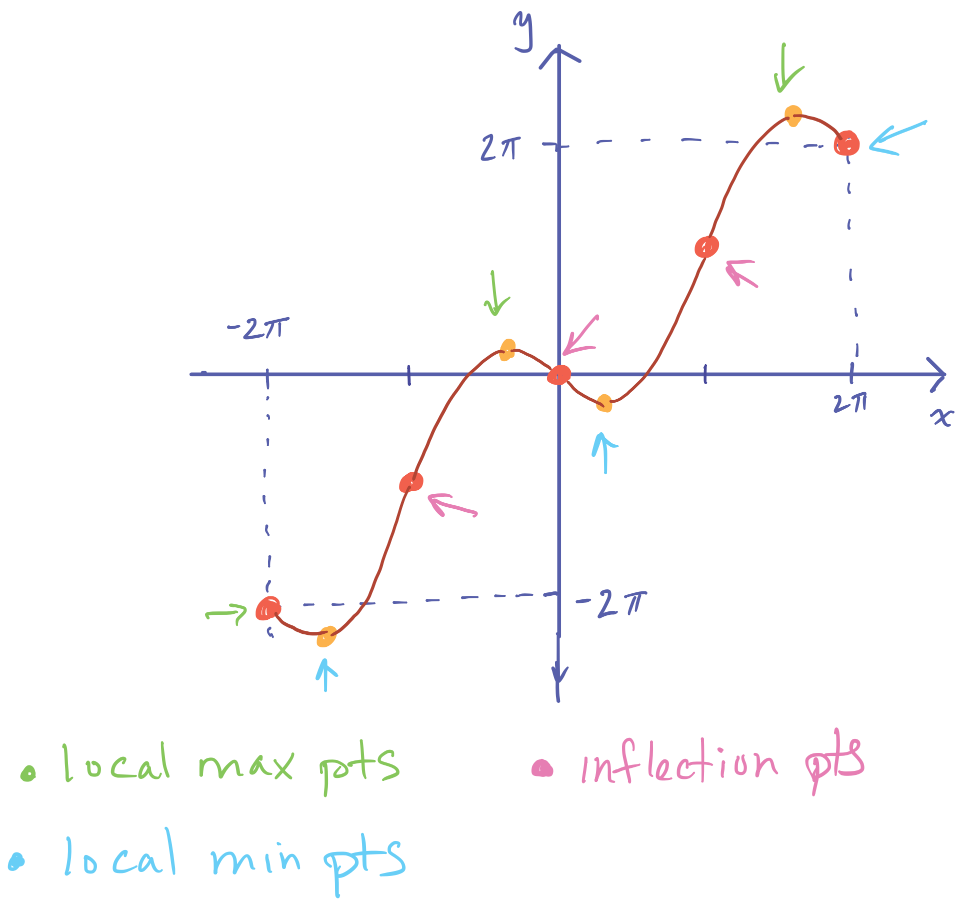
$\left(-\frac{\pi}{3}, \approx 0.69\right)$

$(0, 0)$

$\left(\frac{\pi}{3}, \approx -0.69\right)$

$\left(\frac{5\pi}{3}, \approx 3.5\right)$

$(2\pi, 2\pi)$



2. Sketch the graph of $f(x) = \dfrac{3x^2}{x^2+4}$

(a) Find the domain. $\mathbb{R}$ (denominator never zero!)

(b) Find the x and y-intercepts.

$x=0, y=0.$

(c) Find the symmetries/ periodicity of the curve.

all terms are even. $f(x)$ is even.

(d) Determine the asymptotes.

$y=3$ since $\lim_{x\to\pm\infty} f(x) = 3$. No vertical.

(e,f) Determine where the function is increasing/ decreasing and find the local maximum/ minimum values

$$f'(x) = \frac{24x}{(x^2+4)^2}$$

* details on added page!

$x=0$ only critical pt.

denominator always positive.

$f'>0$ on $(0,\infty)$; $f'<0$ on $(-\infty,0)$

answer:

f is increasing on $(0,\infty)$ and decreasing on $(-\infty,0)$.

f has a local minimum at $x=0$ with minimum value $f(0)=0$.

f has no local maximums.

(g) Find the intervals of concavity/inflection points.

$$f''(x) = \frac{24(4-3x^2)}{(x^2+4)^3}$$

* details on added page

$f''=0$ when $x = \pm 2/\sqrt{3}$

f'' never undefined.

$-$ 0 $+$ 0 $-$

$-2/\sqrt{3}$ $2/\sqrt{3}$

answer

f is concave up on $\left(\frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$ and concave down on $(-\infty, -2/\sqrt{3}) \cup (2/\sqrt{3}, \infty)$.

Inflection points $(-2/\sqrt{3}, 3/4)$ and $(2/\sqrt{3}, 3/4)$

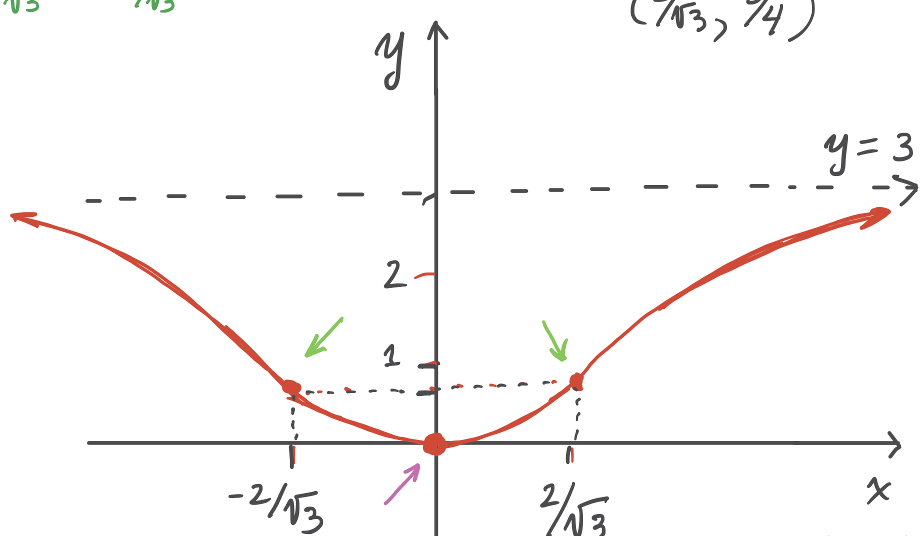
(h) Sketch the curve.

points to plot

$(0,0)$

$\left(-\frac{2}{\sqrt{3}}, \frac{3}{4}\right)$

$\left(\frac{2}{\sqrt{3}}, \frac{3}{4}\right)$



• inflection points

• local min. pt.

3. Sketch the graph of $f(x) = x\sqrt{4-x^2}$

(a) Find the domain.

need $4-x^2 \geq 0$. So $-2 \leq x \leq 2$. ANS: $[-2,2]$

(b) Find the x and y-intercepts.

if $x=0$, $y=0$.

If $y=0$, $x=0,+2,-2$.

(c) Find the symmetries/ periodicity of the curve.

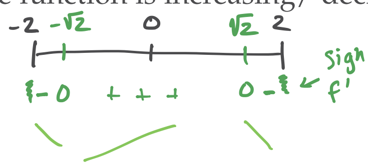
even $\sqrt{4-x^2}$ multiplied by odd x gives odd. $f(x)$ is odd.

(d) Determine the asymptotes.

none

(e,f) Determine where the function is increasing/ decreasing and find the local maximum/ minimum values

$$f'(x) = \frac{2(2-x^2)}{\sqrt{4-x^2}}$$



$f'=0$ when $x=\pm\sqrt{2}$,

f'' undefined at $x=\pm 2$

answer:

f increasing on $(-\sqrt{2},\sqrt{2})$ and decreasing on $(-2,-\sqrt{2})\cup(\sqrt{2},2)$.

f has local min at $x=-\sqrt{2}$, min value -2 and at $x=2$, min value 0.

f has local max at $x=\sqrt{2}$, max value 2 and at $x=-2$, max value 0.

(g) Find the intervals of concavity/inflection points.

$$f''(x) = \frac{2x(6-x^2)}{(4-x^2)^{3/2}}$$

$f''=0$ when $x=0, \cancel{\sqrt{6}}, -\cancel{\sqrt{6}}$ ← not in $[-2,2]$

f'' undefined at $x=-2,2$

$f''<0$ when $x<0$ and $f''>0$ when $x>0$.

answer: f is concave up on $(0,2)$ and concave down on $(-2,0)$.

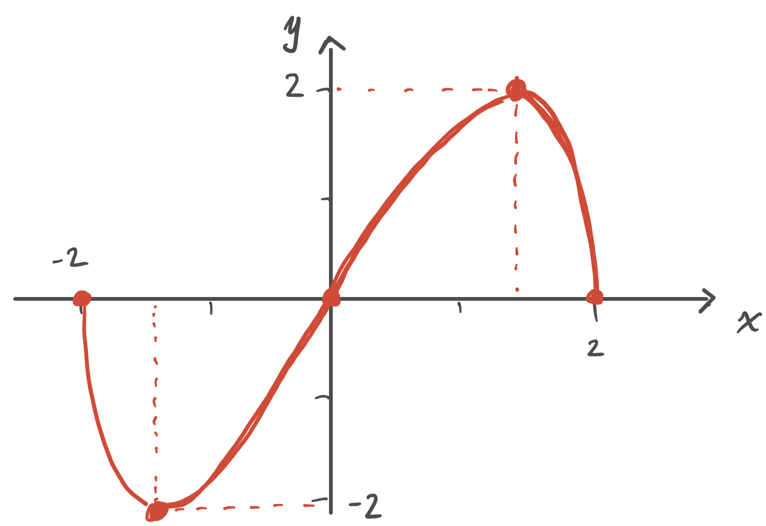
The point $(0,0)$ is an inflection point.

(h) Sketch the curve.

Points to plot

$(-2,0)$, $(0,0)$, $(2,0)$

$(-\sqrt{2},-2)$ $(\sqrt{2},2)$



4. Sketch the curve $y = \dfrac{x}{\sqrt{9+x^2}}$

(a) Find the domain.

$\mathbb{R}$

(b) Find the x and y-intercepts.

$(0,0)$

(c) Find the symmetries/ periodicity of the curve.

odd

(d) Determine the asymptotes. no vertical asymptotes.

$\lim_{x\to\infty} \dfrac{x}{\sqrt{9+x^2}} = 1$. So $y=1$ is a horizontal asym.

tricky! $\lim_{x\to-\infty} \dfrac{x}{\sqrt{9+x^2}} = -1$. So $y=-1$ is a horizontal asymp.

(e,f) Determine where the function is increasing/ decreasing and find the local maximum/ minimum values

$y' = 9(x^2+9)^{-3/2}$

So $y'>0$ always.

answer: y is always increasing.
y has no local max's or mins.

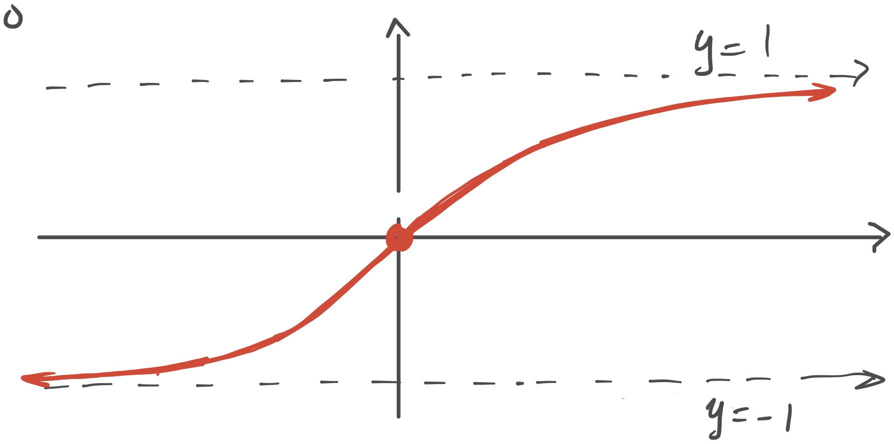
(g) Find the intervals of concavity/inflection points.

$$y'' = \frac{-27x}{(x^2+9)^{5/2}}$$

$y''=0$ when $x=0$.
$y''>0$ when $x<0$; $y''<0$ when $x>0$

answer y is concave up on $(-\infty,0)$ and concave down on $(0,\infty)$.
The point $(0,0)$ is an inflection point.

(h) Sketch the curve.



5. Sketch the curve $y = \dfrac{x^3+4}{x^2}$

(a) Find the domain. $(-\infty, 0) \cup (0, \infty)$

(b) Find the x and y-intercepts.

no y-intercept

Set $y=0$. Then $x = \sqrt[3]{-4} \approx -1.587$

(c) Find the symmetries/ periodicity of the curve. none

the x^3+4 destroys all hope

(d) Determine the asymptotes. (Try to find the slant asymptote. That is, what *line* does this function approach as $x \to \pm\infty$?)

$x=0$ vertical asymptote.

$\lim_{x\to\infty} \frac{x^3+4}{x^2} = \lim_{x\to\infty} x + \frac{4}{x^2}$ which should get closer and closer to $y=x$.

Slant asymptote: $y=x$

(e,f) Determine where the function is increasing/ decreasing and find the local maximum/ minimum values

$+ \quad 0 \quad - \quad 2 \quad +$

$y' = 1 - \frac{8}{x^3}$

$y'=0$ when $x=2$

y' undefined when $x=0$

ans: y is increasing on $(-\infty,0) \cup (2,\infty)$ and decreasing on $(0,2)$

y has a local min at $x=2$ with min value 3

(g) Find the intervals of concavity/inflection points.

$y'' = 24x^{-4} = \frac{24}{x^4}$, which is positive where it is defined.

Ans: y is concave up on $(-\infty,0) \cup (0,\infty)$ with no inflection points.

(h) Sketch the curve.