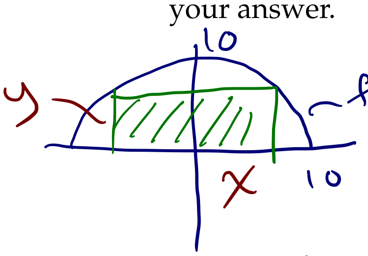
4-7: Optimization (part 2)

1. Find the area of the largest rectangle that can be inscribed in a semicircle of radius 10.
Hint: The radius r of your circle can be considered a fixed constant. You will expect it to appear in your answer. Use coordinates!



$f(x) = \sqrt{100-x^2}$

$A = 2xy$

$A = 2x\sqrt{100-x^2}$ $\quad D: [0, 10]$

$A'(x) = 2\sqrt{100-x^2} + \dfrac{2x\left(\frac{1}{2}\right)(-2x)}{\sqrt{100-x^2}}$

$= \dfrac{2(100-x^2) - 2x^2}{\sqrt{100-x^2}} = \dfrac{4(50-x^2)}{\sqrt{100-x^2}}$

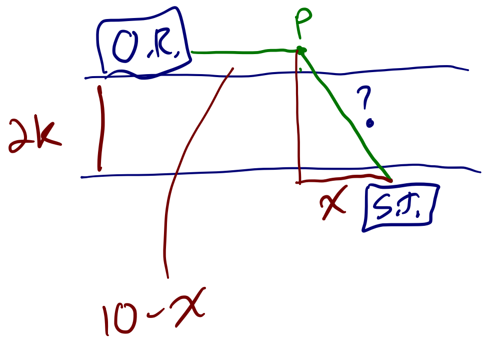
$x = \sqrt{50}$
$= 5\sqrt{2}$

	0	$5\sqrt{2}$	10
A	0	100	0

$y = \sqrt{50} = 5\sqrt{2}$

$\boxed{A = 100 \text{ units}^2}$

2. An oil refinery is located on the north bank of a straight river that is 2 km wide. A pipeline is to be constructed from the refinery to storage tanks located on the south bank of the river 10 km downstream of the refinery. The cost of laying pipe is \$ 10,000/ km over land to a point P on the opposite bank and then \$ 40,000/km under the river to the tanks. To minimize the cost of pipeline, where should P be located?



cost. (in \$10,000's)

$C(x) = 1\cdot(10-x) + 4\left(\sqrt{x^2+4}\right)$

$C'(x) = -1 + \dfrac{2\cdot 2x}{\sqrt{x^2+4}}$

$C'(x) = \dfrac{4x}{\sqrt{x^2+4}} - 1 = 0$

$4x = \sqrt{x^2+4} \Rightarrow 16x^2 = x^2+4$

$x = 2/\sqrt{15}$

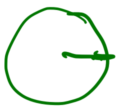
$\boxed{10 - 2/\sqrt{15} \approx 9.48 \text{ km down river from refinery}}$

3. Four feet of wire is used to form a square and a circle. How much of the wire should be used for the square and how much should be used for the circle to enclose the maximum area?

4 ft

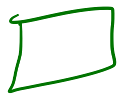
x ↑ circle $\qquad$ $4-x$ ↑ square

Area = area of circle + area of square



circum is x, so $x = 2\pi r$

$r = \frac{x}{2\pi}$



perimeter is $4-x$

$s = \frac{4-x}{4}$

$s = 1 - \frac{x}{4}$

$A(\text{circle}) = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}$

$A(\text{sq.}) = \left(1-\frac{x}{4}\right)^2$

$$A(x) = \frac{x^2}{4\pi} + \left(1-\frac{x}{4}\right)^2$$

$$A'(x) = \frac{x}{2\pi} + 2\left(1-\frac{x}{4}\right)\left(-\frac{1}{4}\right)$$

$$= \frac{x}{2\pi} - \frac{1}{2} + \frac{x}{8} = 0$$

$(\times 8\pi)$

$$4x - 4\pi + \pi x = 0$$

$$(4+\pi)x = 4\pi$$

$$x = \frac{4\pi}{4+\pi}$$

$-$ $\quad$ | $\quad$ $+$

$\frac{4\pi}{4+\pi} \approx 1.76$ ← location of local min!

endpoints:

$x=0$: $A(0) = 1^2 = 1\ ft^2$

$x=4$: $A(4) = \frac{16}{4\pi} + 0 = \boxed{\frac{4}{\pi}\ ft^2}$ maximum

all circle, no square!

$\boxed{\text{SO, all wire for the circle!}}$