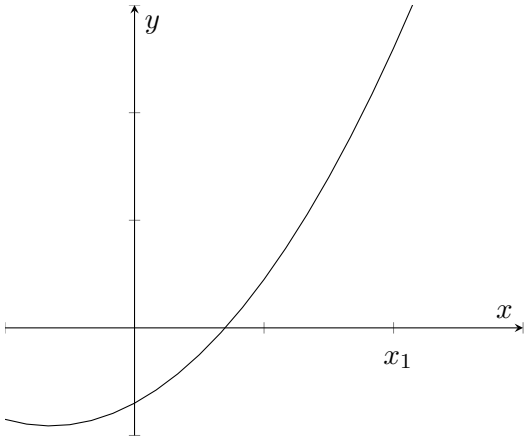
LECTURE NOTES: 4-8 NEWTON'S METHOD

MOTIVATING QUESTION: Suppose we wanted to find the x-intercepts of $f(x) = x - 2\sin x$. From the graph (or the Mean Value Theorem) we can see there exists a positive (and negative) solution. How to find it?

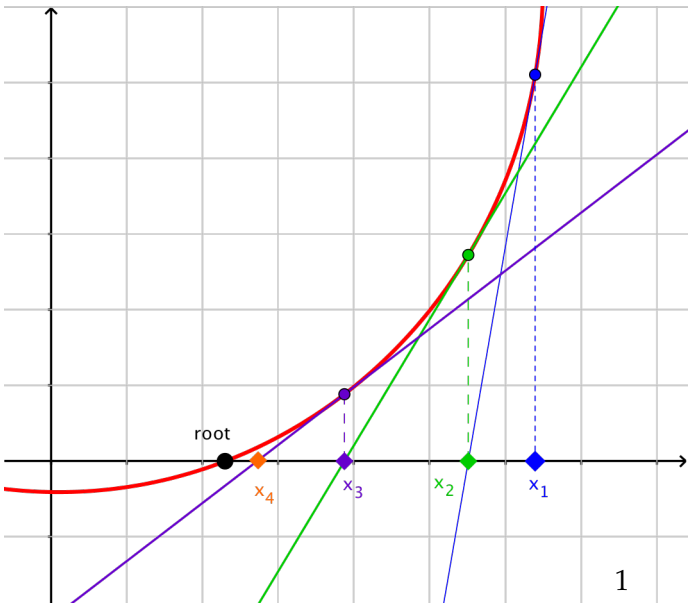
DERIVATION OF NEWTON'S METHOD:

1. Write the equation of the line tangent to the curve $y = f(x)$ at the x-value x_1. Sketch the tangent line in the "cartoon" of $f(x)$ below.



2. In your picture above, label the x-value where the tangent line intersects the x-axis as x_2.
3. Solve for x_2 using your equation from part (1) above.

GEOMETRIC EXPLANATION OF NEWTON'S METHOD:



Formula for Newton's Method:

MODEL PROBLEM: Let $f(x) = x^3 - 5x$.

1. Factor $f(x)$, find its roots algebraically, and sketch its graph.

2. Assume you couldn't factor the function and you wanted to find its positive root. What would be a reasonable first guess and why?

3. Using a first guess of $x_1 = 3$, calculate 2 iterations of Newton's method

4. How close is your estimate of the root, x_3, to the actual root?

5. How important is the first guess (part 2. above)? In particular, are there any truly *bad* guesses that won't get to our sought after root?

EXAMPLE 1: Approximate any zero of $f(x) = x - 2\sin x$ using 2 iterations of Newton's Method. Graph $f(x)$ and draw the first iteration.

EXAMPLE 2: Estimate $\sqrt[6]{7}$ correct to 5 decimal places.