Lecture Notes: Review of Chapters 3 & 4

1. A few warm-up derivatives. Simplify your answers.

   (a) Given $f(x) = \arcsin(e^{3x})$, find $f'(x)$.

   (b) Find dy/dx if $\cos(xy) = x^2 - y$.

   (c) Find y' if $y = \frac{(2x+1)^3}{\sqrt{4+x^2}}$.

   (d) Find $g'(x)$ if $g(x) = (\cos x)^x$.

2. Find the equation of the line tangent to $f(x) = \frac{1+\cos x}{1-\cos x}$ when $x = \pi/2$.

3. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of $2\,\text{cm}^3/\text{sec}$, how fast is the water level rising when the water is 5 cm deep?

4. Find the linearization of $f(x) = \sqrt{x}$ at $a = 4$ and use it to estimate $\sqrt{4.1}$.

5. (a) What are critical numbers of a function f?

(b) How do you find the absolute maximum and minimum of a function f on a closed interval? (Assume f is continuous on the interval.)

(c) Find the critical numbers of $f(x) = \sin x + \cos^2 x$ in $[0, \pi]$.

6. (a) State the Mean Value Theorem and draw a picture to illustrate it.

(b) Determine whether the Mean Value Theorem applies to $f(x) = x(x^2 - x - 2)$ on $[-1, 1]$. If it can be applied find all numbers that satisfy the conclusion of the Mean Value Theorem.

7. Let $f(x) = 2x - 2\cos x$ on $[-\pi, 2\pi]$

   (a) Find the open intervals on which the function is increasing or decreasing.

   (b) Apply the first derivative test to identify all relative extrema. Classify each as a local maxima or local minima.

   (c) Find the open intervals on which the function is concave up or concave down.

   (d) Find the inflection points.

   (e) Sketch the graph.

8. Evaluate the following limits. Show your work.

(a) $\lim_{x\to 0} \frac{1+x-e^x}{\sin x}$

(b) $\lim_{x\to 0^+} (1+2x)^{1/x}$

9. Find the rectangle of maximum area that can be inscribed inside the region bounded above by $y = 20 - x^2$ and bounded below by the x-axis. (Assume the base of the rectangle lies on the x-axis.)