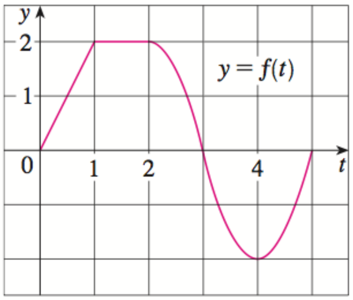
LECTURE: 5-3 THE FUNDAMENTAL THEOREM OF CALCULUS

Example 1: If f is the function whose graph is shown and $g(x) = \int_0^x f(t)dt$, find the values of $g(0)$, $g(1)$, $g(2)$, $g(3)$, $g(4)$ and $g(5)$. Then, sketch a rough graph of g.



The Fundamental Theorem of Calculus, Part 1 If f is continuous on $[a, b]$, the function g defined by

$$g(x) = \int_a^x f(t)dt \quad a \leq x \leq b$$

is continuous on $[a, b]$ and differentiable on (a, b) and $g'(x) = f(x)$.

Example 2: The Fresnel function $S(x) = \int_0^x \sin(\pi t^2/2)dt$ first appeared in Fresnel's theory of the diffraction of light waves. Recently it was applied to the design of highways. Find the derivative of the Fresnel function.

Example 3: Find the derivative of the following functions.

(a) $g(x) = \int_1^{x^4} \sec t dt$

(b) $g(x) = \int_{2x+1}^{2} \sqrt{t} dt$

Example 4: Find the derivative of $g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}} dt$

The Fundamental Theorem of Calculus (Part 2) If f is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is any anti derivative of f, that is, is a function such that $F' = f$

Example 5: Evaluate the following integrals.

(a) $\int_0^1 x^2 dx$

(b) $\int_0^4 (1 + 3y - y^2)dy$

To compute integrals effectively you **must** have your basic anti-differentiation formulas down. You should know that anti-derivatives to the following functions. Note, I'm going to use the $\int$ symbol to mean "find the anti-derivative" of the function right after the symbol.

Anti-Derivatives of Common Functions:

- $\int x^n dx =$ ______________
- $\int \sin x dx =$ ______________
- $\int \cos x dx =$ ______________
- $\int \sec^2 x dx =$ ______________
- $\int \sec x \tan x dx =$ ______________
- $\int \csc^2 x dx =$ ______________
- $\int \csc x \cot x dx =$ ______________
- $\int e^x dx =$ ______________
- $\int a^x dx =$ ______________
- $\int \frac{1}{1+x^2} dx =$ ______________
- $\int \frac{1}{\sqrt{1-x^2}} dx =$ ______________
- $\int \frac{1}{x} du =$ ______________

Example 6: Evaluate the following integrals.

(a) $\int_2^5 \frac{3}{x} dx$

(b) $\int_0^{\pi/2} \cos x dx$

Example 7: Evaluate the following integrals.

(a) $\int_1^8 \sqrt[3]{x} dx$

(b) $\int_{\pi/6}^{\pi/2} \csc x \cot x dx$

(c) $\int_0^1 \frac{9}{1+x^2} dx$

Example 8: We do not have any product or quotient rules for anti-differentiation. To evaluate an integral that is expressed as a product or quotient you must try to manipulate the integrand (the stuff inside the $\int$ sign) to look like something you know how to anti-differentiate. The following integrals are examples of this. Evaluate the following integrals.

(a) $\displaystyle\int_1^3 \frac{x^3 + 3x^6}{x^4}\,dx$

(b) $\displaystyle\int_0^1 x(3 + \sqrt{x})\,dx$

Example 9: Evaluate the following integrals.

(a) $\displaystyle\int_0^2 (5^x + x^5)\,dx$

(b) $\displaystyle\int_{1/2}^{\sqrt{2}/2} \frac{1}{\sqrt{1 - x^2}}\,dx$

Example 10: What is wrong with the following calculation?

$$\int_{-1}^{3} \frac{1}{x^2}\,dx = \frac{x^{-1}}{-1}\Bigg]_{-1}^{3} = -\frac{1}{3} - 1 = -\frac{4}{3}$$