LECTURE: 5-5 THE SUBSTITUTION RULE (PART 1)

Example 1: How would we factor $x^4 - 5x^2 + 6$ and how might it relate to finding $\int 2x\sqrt{1+x^2}\,dx$?

The Substitution Rule If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I then

$$\int f(g(x))g'(x)\,dx = \int f(u)du$$

Note, the substitution rule is basically undoing the ______________ rule.

Example 2: Evaluate $\int x^3 \cos(x^4 + 2)\,dx$ two different ways:

(a) solve for dx.

(b) solve for $x^3\,dx$.

The trickiest thing about subsitution is deciding what to substitute. As substiution is (usually) undoing the chain rule you chould let your u be the inside function. Choose u to be the stuff inside of a power, root sign, denominator, or trigonometric function. When you are choosing your u the derivative of u should appear elsewhere in the integrand up to a constant multiple. The only way to get better is a lot of practice!
Once you make your subsitution the integral usually simplifies considerably. If your original variable does not completely disappear when making the subsitution you either (a) chose a subsitution that doesn't work or (b) made a mistake. At this stage you can try something different, or start your original substitution again.

Example 3: Evaluate the following indefinite integrals.

(a) $\displaystyle\int \sqrt{3x+2}\,dx$

(b) $\int \cos^4 x \sin x\,dx$

Example 4: Evaluate the following indefinite integrals.

(a) $\displaystyle\int \frac{\sec^2 x}{\tan^2 x}\,dx$

(b) $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,dx$

Example 5: Evaluate the following indefinite integrals.

(a) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

(b) $\displaystyle\int \frac{\arctan x}{x^2+1}\,dx$

Example 6: Evaluate the following indefinite integrals.

(a) $\displaystyle\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$

(b) $\displaystyle\int \tan x\,dx$

Example 7: Evaluate the following indefinite integrals.

(a) $\displaystyle\int (1+\tan x)^5 \sec^2 x\,dx$

(b) $\displaystyle\int \frac{\cos(\pi/x)}{x^2}\,dx$

Example 8: Evaluate $\displaystyle\int \frac{5+x}{1+x^2}\,dx$.

Sometimes when you do substitution you also end up solving for your variable in the substitution. For example:
Example 9: Evaluate $\int x^5\sqrt{x^3+1}\,dx$.

Example 10: Evaluate $\int x\sqrt{x+2}\,dx$

Definite Integrals

The Substitution Rule for Definite Integrals: If g' is continuous on $[a,b]$ and f is continuous on the range of $u=g(x)$, then

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

Example 11: Evaluate $\int_0^{\pi/2} \sin^3 x\cos x\,dx$ two ways:

a) going back to x's

b) using substitution