LECTURE: 5-5 THE SUBSTITUTION RULE (PART 2)

Recall:

The Substitution Rule for Definite Integrals: If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

Example 1: Evaluate the following definite integrals.

a) $\displaystyle\int_e^{e^3} \frac{1}{x(\ln x)^2}\,dx$

b) $\displaystyle\int_1^2 x\sqrt{x-1}\,dx$

Example 2: Evaluate the following definite integrals.

a) $\displaystyle\int_0^2 \frac{x}{x^2+4}\,dx$

b) $\displaystyle\int_a^b z^2\cos(1-z^3)\,dz$

Symmetry

- A function f is even if ____________________. Even functions are symmetric about the ____________________.
- A function f is odd if ____________________. Odd functions are symmetric about the ____________________.

Integrals of Even/Odd Functions: Suppose a function $f(x)$ is (blank) on $[-a, a]$. Then,

(a) (even) $\int_{-a}^{a} f(x)\,dx$

(b) (odd) $\int_{-a}^{a} f(x)\,dx$

Example 3: Evaluate the following definite integrals.

(a) $\int_{-2}^{2} (x^2 + 1)\,dx$

(b) $\int_{-1}^{1} \frac{\tan x}{1 + x^2}\,dx$

Example 4: If f is continuous and $\int_{0}^{9} f(x)\,dx = 4$, find $\int_{0}^{3} x f(x^2)\,dx$.

Example 5: Evaluate $\int_{-3}^{3} (x + 5)\sqrt{9 - x^2}\,dx$.

Example 6: Doing (some) substitutions quickly. In later Calculus courses (Calculus 2 especially) it is quite useful to be able to do some very simple substitutions without having to go through writing out u and du. Do the following integrals using substitution and then see if you can see the pattern well enough to not need to do all of the work.

(a) $\displaystyle\int e^{5x}\,dx$ (b) $\displaystyle\int \sin\left(\frac{\pi}{2}x\right)\,dx$ (c) $\displaystyle\int \sqrt{1-2x}\,dx$

Example 7: Integrate the following functions. Check your answers using a derivative.

(a) $\displaystyle\int \sec^2\left(\frac{\pi}{4}\theta\right)\,d\theta$ (b) $\displaystyle\int \sec(2x)\tan(2x)\,dx$ (c) $\displaystyle\int \sqrt{1+4x}\,dx$

Example 8: Evaluate the following integrals.

(a) $\displaystyle\int xe^{-x^2}\,dx$ (b) $\displaystyle\int_1^4 \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx$