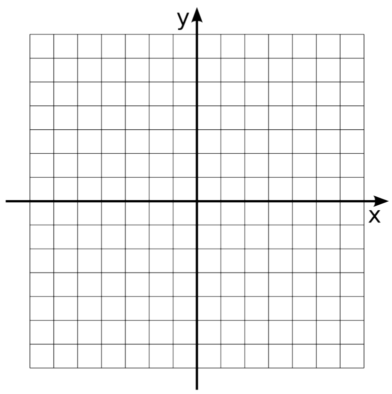
Final Review - Chapter 2
(Limits, + Continuity + L'Hospital's Rule)

Example 1: Sketch the graph of $f(x) = \begin{cases} \sqrt{-x}, & \text{if } x < 0 \\ x^2 \text{ if } 0 < x \leq 2 \\ x - 5, & \text{if } x > 2 \end{cases}$ and give the interval on which f is continuous. At what numbers is f continuous from the right, left or neither?



a) $\lim_{x \to 0^-} f(x)$

b) $\lim_{x \to 0^+} f(x)$

c) $\lim_{x \to 0} f(x)$

d) $\lim_{x \to 2^-} f(x)$

e) $\lim_{x \to 2^+} f(x)$

f) $\lim_{x \to 2} f(x)$

- Find limits using factoring, algebra, conjugates.

Example 2: Find the following limits:

a) $\lim_{x \to -1^-} f(x)$ for $f(x) = \begin{cases} x^2 - 1 & \text{for } x < 1 \\ 2x + 3 & \text{for } x \geq 1 \end{cases}$

b) $\lim_{x \to 1} e^{x-1} \sin\left(\frac{\pi x}{2}\right)$

c) $\lim_{x \to 0} \frac{5x^2}{1 - \cos x}$

Example 3: Find the following limits:

a) $\lim_{x \to 3} \dfrac{2x^2 - 18}{x^2 + x - 12}$

b) $\lim_{h \to 0} \dfrac{(4+h)^3 - 64}{h}$

c) $\lim_{x \to -4} \dfrac{\frac{1}{4} + \frac{1}{x}}{4 + x}$

- Find infinite limits. As in the limit is equal to plus or minus infinity or has an infinite discontinuity.
- Find limits at infinity. This means x goes to plus or minus infinity.

Example 4: Find the following limits:

a) $\lim_{x \to 5^-} \dfrac{e^x}{(x-5)^3}$

b) $\lim_{x \to \pi^-} \cot x$

Example 5: Find the following limits.

a) $\displaystyle\lim_{x\to\infty} \frac{4x^4+5}{(x^2-2)(2x^2-1)}$

b) $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{9x^6-x}}{x^3+1}$

Example 6: Find the following limits.

a) $\displaystyle\lim_{x\to\infty} \sec\left(\frac{x^2}{x^3-2}\right)$

b) $\displaystyle\lim_{x\to 0^+} \arctan(1/x)$

Example 7: Find the following limits using l'Hospital's rule. I won't tell you explicitly to do this on the exam. You will have to know when you can/ cannot apply this rule.

a) $\displaystyle\lim_{x\to\infty} \frac{1-e^x}{1+2e^x}$

b) $\displaystyle\lim_{h\to 0} \frac{\sin h}{h\cos h}$

- Know and apply the defintion of continuity.
- Determine where a function is discontinuous and why. continuous.

Definition of Continuity A function f is continuous at c if the following three conditions are met:

1. ______________________________

2. ______________________________

3. ______________________________

Example 8: Find all points of discontinuity of $h(x) = \dfrac{x-4}{x^2-x-12}$ and explain why the points are discontinuous and state if they are removable or non-removable.

Final Review - Chapter 3 (Derivative rules)

- Find derivatives using the limit defintion.
- Know how to apply the sum, difference, product, quotient, and chain rules.
- Know when to use logarithmic differentiation to find a derivative.

Example 1: Find the derivative of $f(x) = 9 + x - 2x^2$ using the definition of the derivative. Then find an equation of the tangent line at the point $(2, 3)$.

Example 2: Calculate y'.

a) $y = \dfrac{1}{\sqrt{x}} - \dfrac{1}{\sqrt[5]{x^3}}$

b) $y = \dfrac{\tan x}{1 + \cos x}$

Example 3: Calculate y'.

a) $y = x \cos^{-1} x$

b) $y = (\arcsin(2x))^2$

Example 4: Calculate y'.

a) $y = e^{x \sec x}$

b) $y = 10^{\tan(\pi\theta)}$

Example 5: Find $\frac{dy}{dx}$.

a) $y = \arcsin(e^{2x})$

b) $y = \displaystyle\int_{x^2}^{3} \frac{t+4}{\cos t}\,dt$

Example 6: Find the derivative of $h(x) = \ln\left(\dfrac{x^2-4}{2x+5}\right)$

- Solve related rates problems.

Example 7: The sides of an equilateral triangle are increasing at a rate of 10 cm/min. At what rate is the area of the triangle increasing when the sides are 30 cm long? ($A = \frac{\sqrt{3}}{4}(\text{side})^2$)

Example 8: The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2?