Final Review - Chapter 4
(Applications of Differentiation)

- Find critical numbers of a function.
- Find the absolute maximum and absolute minimum of a function on a closed interval.

Example 1: Find the absolute maximum and minimum of $f(x) = xe^{x/2}$ on $[-3, 1]$

- Determine where a function is increasing decreasing.
- Determine where a function is concave up and concave down.

Example 2: Given $G(x) = 5x^{2/3} - 2x^{5/3}$

(a) Find the intervals of increase/ decrease.

(b) Find the local maximum and minimum values.

(c) Find the intervals of concavity and the inflection points.

- Solve max/ min optimization problems.

Example 3: A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

Example 4: Suppose a box with a square base and open top must have a volume of 32 m^3. Find the dimensions of the box that minimize the amount of material used.

Example 5: A rectangular storage container with an open top is to have a volume of 10 m^3. The length of the base is twice the width. Material for the base costs $10 per square meter. Material for the sides costs $ 6 per square meter. Find the costs of materials for the cheapest such container.

- Apply Newton's method to take a "step" (get a better approximation of a root of a function.)

Example 6: Use one iteration of Newton's method with $x_1 = -1$ to get a better approximation of the root of $f(x) = x^7 + 4$. [I.e., find x_2.] After that, graph $f(x)$ and demonstrate how x_2 was obtained from x_1.

Final Review - Chapter 5
(Integration)

Example 1: Find the most general antiderivative of the function.

a) $g(x) = \dfrac{1}{x} + \dfrac{1}{x^2+1}$

b) $f(x) = \dfrac{x^2+\sqrt{x}}{x}$

Example 2: A particle is moving with $v(t) = 2t - 1/(1+t^2)$ and $s(0) = 1$. Find the position of the particle.

Example 3: Compare/contrast the applications of FTC below.

a) Find the derivative of
$$g(x) = \int_1^{x^2} t^3\sqrt{1+t^4}\,dt$$

b) Evaluate $\displaystyle\int_1^a t^3\sqrt{1+t^4}\,dt$

Example 4: Estimate the area under the curve $y = x^2 + 2$ on the interval $[0, 8]$ using 4 sub-intervals and the method given below.

a) left endpoints.

b) midpoints.

Example 5: Evaluate the following definite integrals.

a) $\displaystyle\int_0^{\pi/4} \frac{\sec^2 t}{\tan t + 1}\,dt$

b) $\displaystyle\int_1^4 \frac{x-2}{\sqrt{x}}\,dx$

Example 6: Evaluate the following indefinite integrals.

a) $\displaystyle\int \frac{\sin(1/x)}{x^2}\,dx$

b) $\displaystyle\int \frac{x}{(x-2)^3}\,dx$

Example 7: A particle moves along a line with velocity function $v(t) = 2\sin t$, where v is measured in meters per second.

(a) Find the displacement over the time interval $[0, 6]$

(b) Find the total distance traveled during the time interval $[0, 6]$

Example 8: A bacteria population is 4000 at time $t = 0$ and its rate of growth is 1000×2^t bacteria per hour after t hours. What is the population after one hour?