Your Name

Your Signature

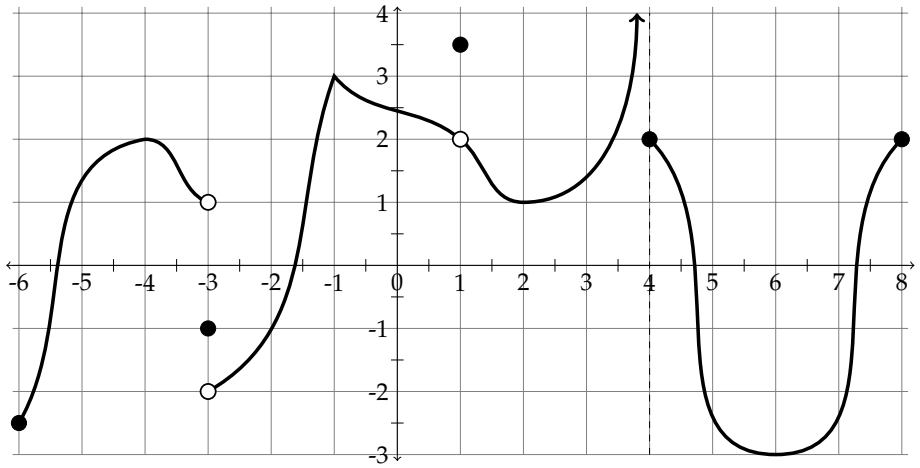
Instructor Name

End Time

Problem	Total Points	Score
1	10	
2	8	
3	8	
4	15	
5	6	
6	6	
7	16	
8	20	
9	5	
10	6	
Extra Credit	(5)	
Total	100	

- The total time allowed for this exam is two hours.
- This test is closed notes and closed book.
- You may **not** use a calculator.
- In order to receive full credit, you must **show your work.** Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- **PLACE A BOX AROUND** $\boxed{\textbf{YOUR FINAL ANSWER}}$ **to each question** where appropriate.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

1 (10 points) The graph of the function $f(x)$ is given below. Use it to answer the questions below. If you are asked to determine a limit, find the limit or one-sided limit as directed. Use ∞ and $-\infty$ where appropriate. If the limit does not exist and cannot be described using ∞ or $-\infty$, write "DNE".



(a) $\lim_{x \to -3^-} f(x) =$

(b) $\lim_{x \to 1} f(x) =$

(c) $\lim_{x \to -3} f(x) =$

(d) $\lim_{x \to 4^-} f(x) =$

(e) At what x-values in its domain is $f(x)$ NOT continuous? If f is continuous everywhere on its domain, write "none".

(f) At what x-values in its domain is $f(x)$ NOT differentiable? If f is differentiable everywhere on its domain, write "none".

(g) What are the x-values corresponding to local maxima of $f(x)$? If there aren't any, write "none".

(h) What are the x-values corresponding to local minima of $f(x)$? If there aren't any, write "none".

(i) What are the x-values corresponding to absolute maxima of $f(x)$? If there aren't any, write "none".

(j) What are the x-values corresponding to absolute minima of $f(x)$?If there aren't any, write "none".

2 (8 points)

(a) Complete the definition of the derivative of a function $f(x)$ below:

$$f'(x) =$$

(b) Find the derivative of $f(x) = 5x^2 - x$ using the definition of the derivative. You must show your work to receive credit.

3 (8 points) The volume of a circular cylinder is increasing at a rate of 20π m^3/sec while the radius is increasing at a rate of 2 m/sec. How must the height of the cylinder be changing when the volume is 90π m^3 and the radius is 3 m? Include units with your answer.

4 (15 points) Calculate the derivatives of the given functions.

(a) $y = (x^2 + 1)^{\cos x}$

(b) $g(z) = \dfrac{\sec(8z)}{1 + z^2}$

(c) $h(x) = \displaystyle\int_{\arctan x}^{5} \sqrt{3 + 2t^3}\, dt$

5 (6 points) Let $f(x) = e^{4x} \cos x$.

(a) (4 points) Find the linearization of the function $f(x)$ at the point $a = 0$.

(b) (2 points)Use your linear approximation from part (a) to estimate $f(0.1)$.

6 (6 points) Find the absolute maximum and absolute minimum of $f(x) = x^3 - 3x + 5$ on the interval $[0, 3]$.

7 (16 points) Answer the following questions using the given function and its derivatives. **Note that this problem continues onto the next page.**

$$f(x) = \frac{3x^2 - 1}{x^3}, \qquad f'(x) = \frac{-3(x^2 - 1)}{x^4}, \qquad f''(x) = \frac{6(x^2 - 2)}{x^5}$$

(a) Find the vertical asymptotes, if any.

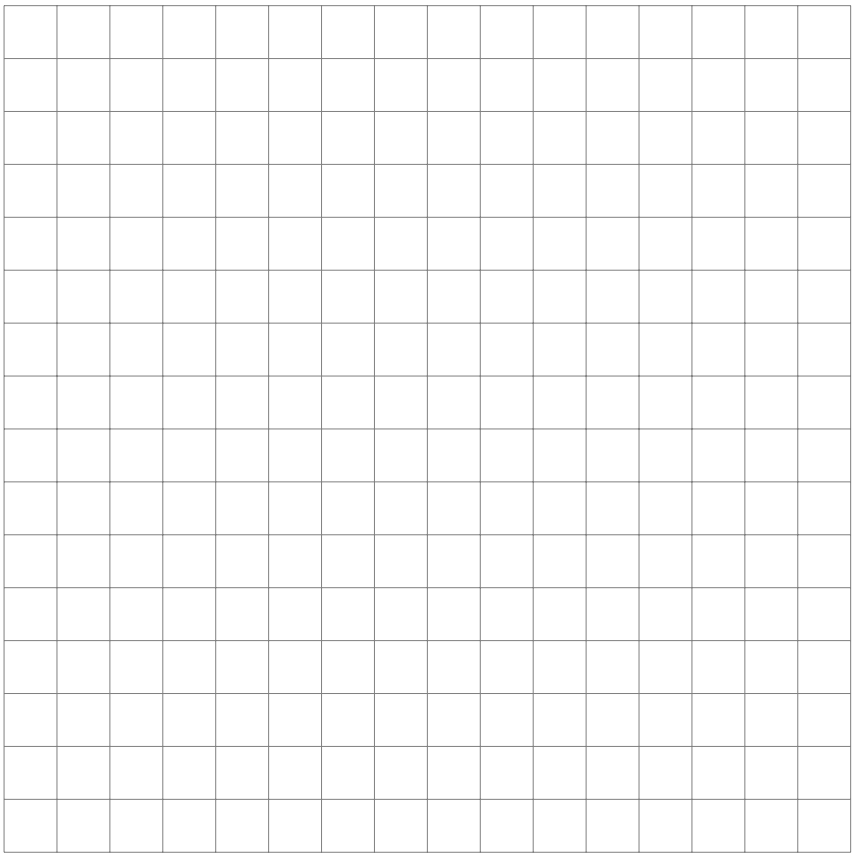
(b) Find the horizontal asymptotes, if any.

(c) Find the intervals of increase or decrease.

(d) Find the local maximum and minimum values, if any.

(e) Find the intervals of concavity and the inflection points.

(f) Use the information from parts (a) - (e) to sketch the graph.



8 (20 points) Evaluate the following integrals.

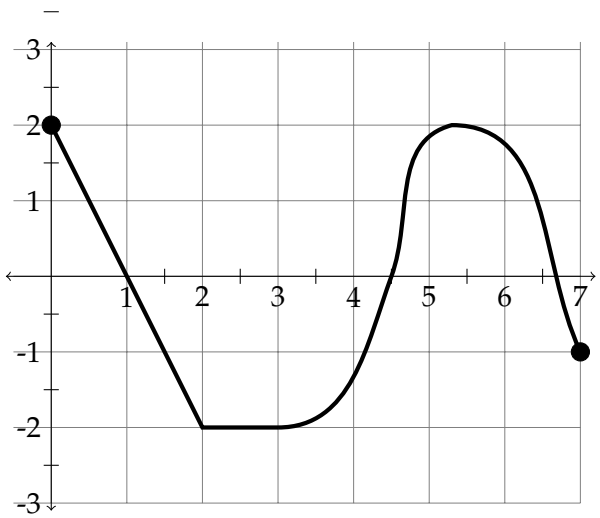
(a) $\displaystyle\int \frac{1+x^2}{x^{5/3}}dx$

(b) $\displaystyle\int \frac{x\sin(x^2)}{8}dx$

(c) $\displaystyle\int_e^{e^4} \frac{4\,dx}{x(\ln x)^3}$

(d) $\displaystyle\int 2x\sqrt{x+5}\,dx$

9 (5 points) The graph of $f(x)$ is given below.



(a) Evaluate $\int_0^1 f(x)\,dx$.

(b) Evaluate $\int_0^3 f(x)\,dx$.

(c) Where does $g(x) = \int_0^x f(t)\,dt$ achieve a local maximum on the interval $(0, 7)$? Justify your answer.

(d) Where does $g(x) = \int_0^x f(t)\,dt$ achieve a local minimum on the interval $(0, 7)$? Justify your answer.

(e) Are there any values of x such that $g(x) = \int_0^x f(t)\,dt = 0$ on $[0, 7]$? Justify your answer.

10 (6 points) Water flows into a reservoir at a rate of $1000 - 20t$ liters per hour.

(a) What does the quantity $\int_1^5 (1000 - 20t)\, dt$ represent?

(b) Assume the reservoir initially contained 50,000 liters, how much water is in the reservoir after 4 hours?

11 (5 points) **[Extra Credit]** An offshore oil well is 2 kilometers off the coast. The refinery is 4 kilometers down the coast. Laying pipe in the ocean is twice as expensive as on land. What path should the pipe follow in order to minimize the cost?