Your Name

Beth Zirbes

Your Signature

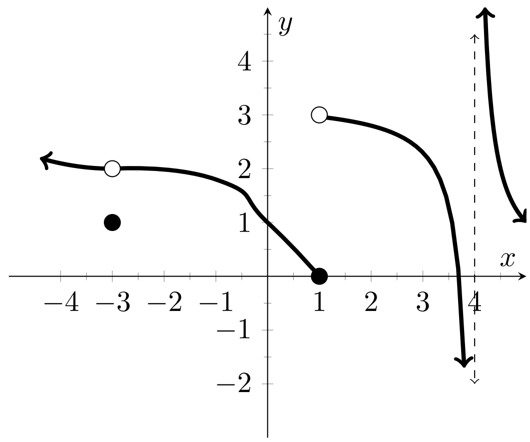
Instructor Name

End Time

Problem	Total Points	Score
1	8	
2	10	
3	18	
4	18	
5	10	
6	8	
7	8	
8	6	
9	10	
10	4	
Extra Credit	(5)	
Total	100	

- This test is closed notes and closed book.
- You may **not** use a calculator.
- In order to receive full credit, you must **show your work.** Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- **PLACE A BOX AROUND** **YOUR FINAL ANSWER** **to each question** where appropriate.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

$\boxed{1}$ (8 points) For the function $f(x)$ whose graph is given below, state the value of each quantity if it exists.



(a) $\lim_{x\to -3} f(x) =$ 2

(b) $f(-3) =$ 1

(c) $\lim_{x\to 1^-} f(x) =$ 0

(d) $\lim_{x\to 1^+} f(x) =$ 3

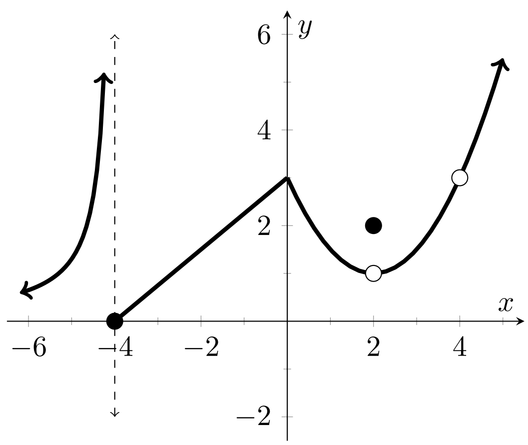
(e) $\lim_{x\to 1} f(x) =$ DNE

(f) $f(1) =$ 0

(g) $\lim_{x\to 4^-} f(x) =$ $-\infty$

(h) $\lim_{x\to 4^+} f(x) =$ ∞

$\boxed{2}$ (10 points) A graph of the function $f(x)$ is displayed below.



(a) (6 points) From the graph of f, state the numbers at which f is discontinuous and why.

at $x=-4$ because $\lim_{x\to -4} f(x)$ DNE (left & right limits are unequal.)

at $x=2$ because $\lim_{x\to 2} f(x) \neq f(2)$

at $x=4$ because $f(4)$ is not defined

(b) (4 points) From the graph of f, state the numbers at which f fails to be differentiable and why.

at $x=-4, 2, 4$ as f is not continuous

at $x=0$ as the graph has a corner/cusp.

3 (18 points) Evaluate the following limits. Justify your answers with words and/ or any relevent algebra. Be sure to use proper notation, as points will be deducted for not doing so.

(a) $\lim_{x\to -3} \frac{x^2+3x}{x^2-x-12} = \lim_{x\to -3} \frac{x(x+3)}{(x+3)(x-4)}$

$= \lim_{x\to -3} \frac{x}{(x-4)}$

$= \frac{-3}{-7}$

$= \boxed{\frac{3}{7}}$

(b) $\lim_{x\to 1} \ln\left(\frac{5-x^2}{1+x}\right) = \ln\left(\lim_{x\to 1} \frac{5-x^2}{1+x}\right)$

$= \ln\left(\frac{5-1}{1+1}\right)$

$= \ln\left(\frac{4}{2}\right)$

$= \boxed{\ln(2)}$

(c) $\lim_{x\to\infty} \frac{(1-x^2)\,1/x^2}{(x^3-x+1)\,1/x^2} = \lim_{x\to\infty} \frac{(1/x^2-1)}{(x-1/x+1/x^2)}$

$= \frac{-1}{\infty}$

$= \boxed{0}$

4 (18 points) Evaluate the following limits. Justify your answers with words and/ or any relevent algebra. Be sure to use proper notation, as points will be deducted for not doing so.

(a) $\lim_{x\to 4^-} \frac{\sqrt{x}}{(x-4)^5} = \boxed{-\infty}$

As $x \to 4^-$, the numerator approaches $\sqrt{4} = 2$, a positive constant.

As $x \to 4^-$, the denominator approaches zero and is negative.

A positive constant divided by a small negative results in negative infinity.

(b) $\lim_{x\to 3} \frac{\frac{1}{x^2} - \frac{1}{9}}{x-3} = \lim_{x\to 3} \frac{(9-x^2)}{9x^2} \cdot \frac{1}{(x-3)} \qquad = \boxed{-\frac{2}{27}}$

$= \lim_{x\to 3} \frac{(3+x)(3-x)}{9x^2(x-3)}$

$= \lim_{x\to 3} -\frac{(3+x)}{9x^2}$

$= \frac{-6}{81}$

(c) $\lim_{x\to -\infty} \frac{\sqrt{1+4x^6}}{2-x^3} = \lim_{x\to\infty} \frac{\sqrt{1+4(-x)^6}}{2-(-x)^3}$

$= \lim_{x\to\infty} \frac{\sqrt{1+4x^6}}{(2+x^3)} \begin{matrix} 1/x^3 \\ 1/x^3 \end{matrix}$

$= \lim_{x\to\infty} \frac{\sqrt{1/x^6 + 4}}{2/x^3 + 1}$

$= \frac{\sqrt{0+4}}{0+1}$

$= \boxed{2}$

5 (10 points) Given $f(x) = \begin{cases} 3 & x \geq 4 \\ \frac{3x-12}{|x-4|} & x < 4 \end{cases}$ find $\lim_{x\to 4} f(x)$ or explain why this limit does not exist.

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} \frac{3x-12}{|x-4|}$$
$$= \lim_{x\to 4^-} \frac{3(x-4)}{-(x-4)}$$
$$= \lim_{x\to 4^-} (-3)$$
$$= -3$$

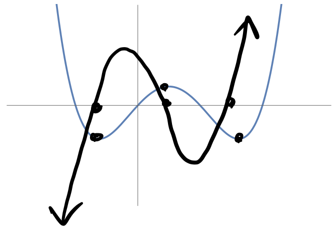
$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} 3$$
$$= 3$$

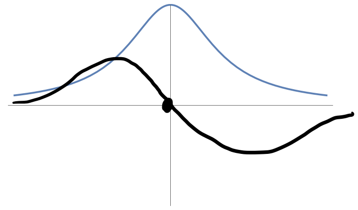
As $\lim_{x\to 4^-} f(x) \neq \lim_{x\to 4^+} f(x)$, the $\lim_{x\to 4} f(x)$ does not exist.

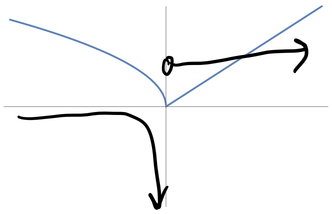
6 (8 points) Using complete sentences, use the Interemdiate Value Theorem to show that there is a root of the equation $e^x = 3 - 2x$ in the interval $(0, 1)$.

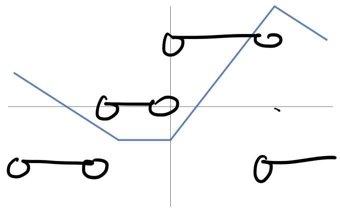
Note $e^x = 3-2x$ implies $e^x + 2x - 3 = 0$.
Let $f(x) = e^x + 2x - 3$, a continuous function.
Notice $f(0) = e^0 + 2(0) - 3 = -2 < 0$ and
$f(1) = e + 2 - 3 \approx 4.71 - 3 > 0$.
As $f(0) < 0$ and $f(1) > 0$, and f is continuous,
we know $f(c) = 0$ for some c in $(0,1)$. Thus,
$e^x = 3-2x$ has a root in $(0,1)$.

$\boxed{7}$ (8 points) Match the graph of each function (a) - (d) with the graph of its derivative I-VIII. Please put your answers in the blanks provided below the graphs.

a.) 

c.) 

b.) 

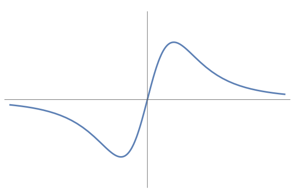
d.) 

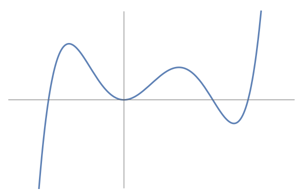
(a) Graph (a)'s derivative is given by VII

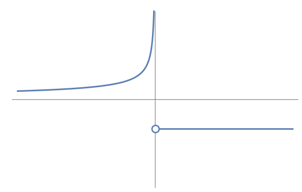
(b) Graph (b)'s derivative is given by V

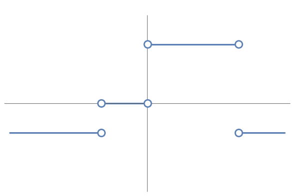
(c) Graph (c)'s derivative is given by VIII

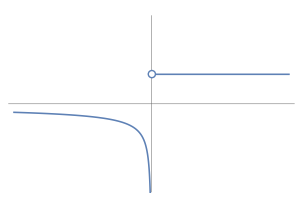
(d) Graph (d)'s derivative is given by IV

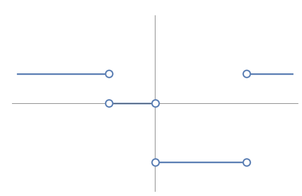
I. 

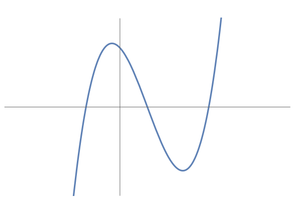
II. 

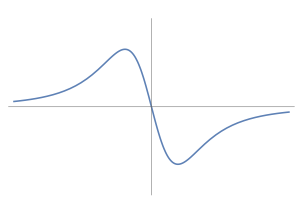
III. 

IV. 

V. 

VI. 

VII. 

VIII. 

8 (6 points) Given $f(x) = \dfrac{3}{x}$ the derivative of $f(x)$ is given by $f'(x) = -\dfrac{3}{x^2}$. Using this derivative find the equation of the tangent line to $f(x)$ when $x = 3$. Give your final answer in slope-intercept form.

point: $f(3) = 3/3 = 1$ or $(3,1)$

slope: $m = f'(3) = -3/9 = -1/3$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -1/3 (x - 3)$$

$$y - 1 = -1/3 x + 1$$

$$\boxed{y = -1/3 x + 2}$$

9 (10 points)

(a) (2 points) State the limit definition of the derivative of $f(x)$.

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

+1

(b) (8 points) Given $f(x) = \sqrt{3x}$, find $f'(x)$ using the definition. **No credit will be given for answers found using derivative short-cut formulas**. Simplify your final answer.

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{3(x+h)} - \sqrt{3x}}{h}$$

$$= \lim_{h \to 0} \left(\frac{\sqrt{3x+3h} - \sqrt{3x}}{h}\right)\left(\frac{\sqrt{3x+3h} + \sqrt{3x}}{\sqrt{3x+3h} + \sqrt{3x}}\right)$$

$$= \lim_{h \to 0} \frac{3x + 3h - 3x}{h(\sqrt{3x+3h} + \sqrt{3x})}$$

$$= \lim_{h \to 0} \frac{3h}{h(\sqrt{3x+3h} + \sqrt{3x})}$$

$$= \lim_{h \to 0} \frac{3}{\sqrt{3x+3h} + \sqrt{3x}}$$

$$= \frac{3}{\sqrt{3x} + \sqrt{3x}}$$

$$= \boxed{\frac{3}{2\sqrt{3x}}}$$

$\boxed{10}$ (4 points) The number of bacteria after t hours in a controlled laboratory setting is given by the function $n = f(t)$ where n is the number of bacteria and t is measured in hours.

(a) Suppose $f'(5) = 2000$. What are the units of the derivative?

The units are $\boxed{\text{number of bacteria / hour}}$

or $\boxed{\text{number of bacteria per hour}}$

(b) In the context of this situation, explain what $f'(5) = 2000$ means using complete sentences.

At $t = 5$ hours, the population of bacteria is growing at a rate of 2000 bacteria per hour.

$\boxed{11}$ (5 points) **Extra Credit:** Prove that $\lim\limits_{x\to 0} x^4 \cos\dfrac{2}{x} = 0$. You must clearly explain your work and cite any relevent theorems for full credit.

Note $-1 \leq \cos\left(\frac{2}{x}\right) \leq 1$.

As x^4 is positive, $-x^4 \leq x^4 \cos\left(\frac{2}{x}\right) \leq x^4$.

Since $\lim\limits_{x\to 0}(-x^4) = 0$ and $\lim\limits_{x\to 0} x^4 = 0$,

$\lim\limits_{x\to 0} x^4 \cos\left(\frac{2}{x}\right) = 0$ by the squeeze theorem.