Your Name

Your Signature

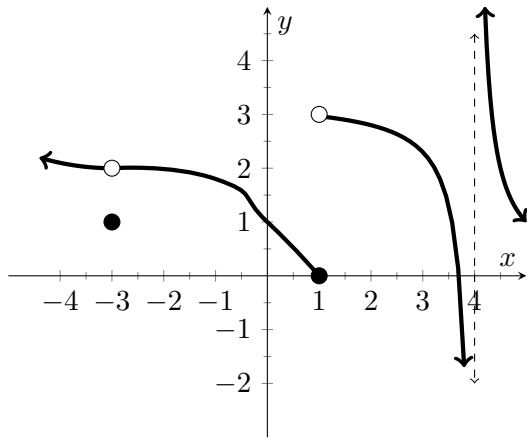
Instructor Name

End Time

Problem	Total Points	Score
1	8	
2	10	
3	18	
4	18	
5	10	
6	8	
7	8	
8	6	
9	10	
10	4	
Extra Credit	(5)	
Total	100	

- The total time allowed for this exam is 60 minutes.
- This test is closed notes and closed book.
- You may **not** use a calculator.
- In order to receive full credit, you must **show your work.** Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- **PLACE A BOX AROUND** $\boxed{\textbf{YOUR FINAL ANSWER}}$ **to each question** where appropriate.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

1 (8 points) For the function $f(x)$ whose graph is given below, state the value of each quantity if it exists.



(a) $\lim_{x\to -3} f(x) =$ _____

(b) $f(-3) =$ _____

(c) $\lim_{x\to 1^-} f(x) =$ _____

(d) $\lim_{x\to 1^+} f(x) =$ _____

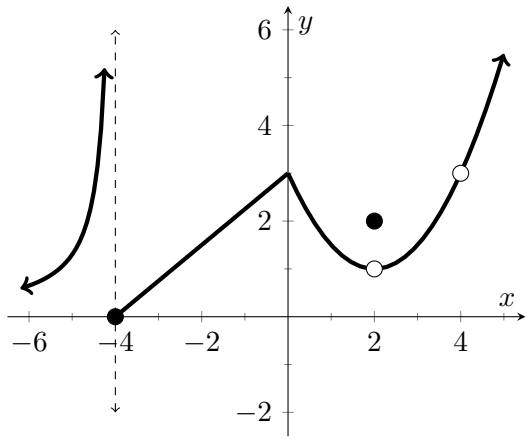
(e) $\lim_{x\to 1} f(x) =$ _____

(f) $f(1) =$ _____

(g) $\lim_{x\to 4^-} f(x) =$ _____

(h) $\lim_{x\to 4^+} f(x) =$ _____

2 (10 points) A graph of the function $f(x)$ is displayed below.



(a) (6 points) From the graph of f, state the numbers at which f is discontinuous and why.

(b) (4 points) From the graph of f, state the numbers at which f fails to be differentiable and why.

3 (18 points) Evaluate the following limits. Justify your answers with words and/ or any relevent algebra. Be sure to use proper notation, as points will be deducted for not doing so.

(a) $\lim_{x \to -3} \frac{x^2 + 3x}{x^2 - x - 12}$

(b) $\lim_{x \to 1} \ln\left(\frac{5 - x^2}{1 + x}\right)$

(c) $\lim_{x \to \infty} \frac{1 - x^2}{x^3 - x + 1}$

$\boxed{4}$ (18 points) Evaluate the following limits. Justify your answers with words and/ or any relevent algebra. Be sure to use proper notation, as points will be deducted for not doing so.

(a) $\lim_{x \to 4^-} \frac{\sqrt{x}}{(x-4)^5}$

(b) $\lim_{x \to 3} \frac{\frac{1}{x^2} - \frac{1}{9}}{x-3}$

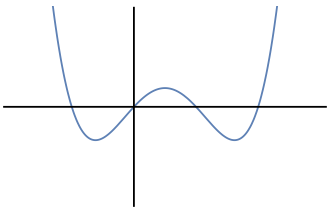
(c) $\lim_{x \to -\infty} \frac{\sqrt{1+4x^6}}{2-x^3}$

$\boxed{5}$ (10 points) Given $f(x) = \begin{cases} 3 & x \geq 4 \\ \frac{3x-12}{|x-4|} & x < 4 \end{cases}$ find $\lim_{x \to 4} f(x)$ or explain why this limit does not exist.

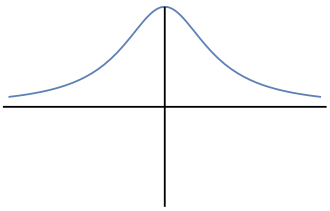
$\boxed{6}$ (8 points) Using complete sentences, use the Interemdiate Value Theorem to show that there is a root of the equation $e^x = 3 - 2x$ in the interval $(0, 1)$.

7 (8 points) Match the graph of each function (a) - (d) with the graph of its derivative I-VIII. Please put your answers in the blanks provided below the graphs.

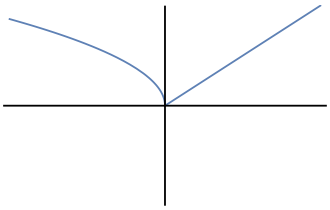
a.)



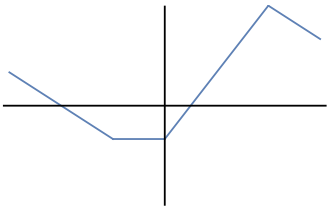
c.)



b.)



d.)



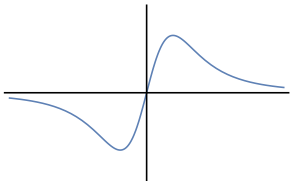
(a) Graph (a)'s derivative is given by _____

(b) Graph (b)'s derivative is given by _____

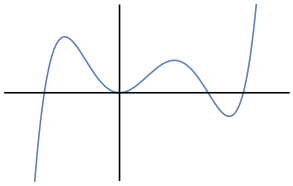
(c) Graph (c)'s derivative is given by _____

(d) Graph (d)'s derivative is given by _____

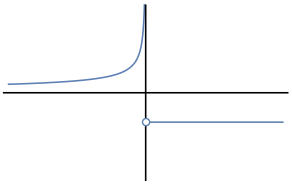
I.



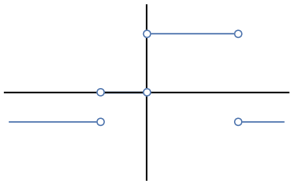
II.



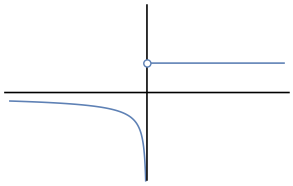
III.



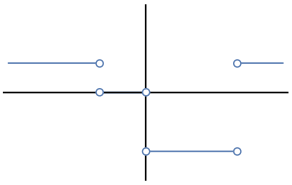
IV.



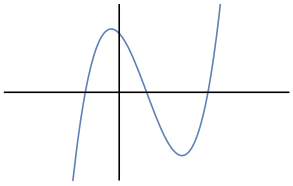
V.



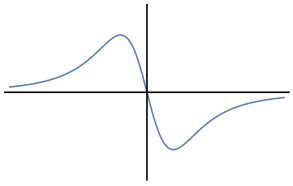
VI.



VII.



VIII.



8 (6 points) Given $f(x) = \frac{3}{x}$ the derivative of $f(x)$ is given by $f'(x) = -\frac{3}{x^2}$. Using this derivative find the equation of the tangent line to $f(x)$ when $x = 3$. Give your final answer in slope-intercept form.

9 (10 points)

(a) (2 points) State the limit definition of the derivative of $f(x)$.

(b) (8 points) Given $f(x) = \sqrt{3x}$, find $f'(x)$ using the definition. **No credit will be given for answers found using derivative short-cut formulas**. Simplify your derivative.

$\boxed{10}$ (4 points) The number of bacteria after t hours in a controlled laboratory setting is given by the function $n = f(t)$ where n is the number of bacteria and t is measured in hours.

(a) Suppose $f'(5) = 2000$. What are the units of the derivative?

(b) In the context of this situation, explain what $f'(5) = 2000$ means using complete sentences.

$\boxed{11}$ (5 points) **Extra Credit:** Prove that $\lim_{x \to 0} x^4 \cos \frac{2}{x} = 0$. You must clearly explain your work and cite any relevent theorems for full credit.