Your Name

Beth Zirbes

Your Signature

Instructor Name

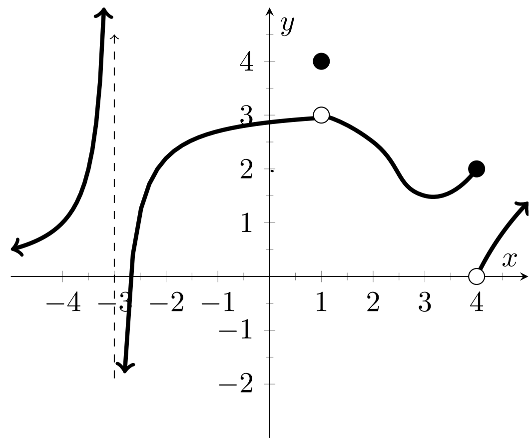
End Time

Problem	Total Points	Score
1	8	
2	10	
3	18	
4	18	
5	10	
6	8	
7	8	
8	6	
9	10	
10	4	
Extra Credit	(5)	
Total	100	

- This test is closed notes and closed book.
- You may **not** use a calculator.
- In order to receive full credit, you must **show your work.** Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- **PLACE A BOX AROUND** $\boxed{\textbf{YOUR FINAL ANSWER}}$ **to each question** where appropriate.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

$\boxed{1}$ (8 points)

For the function $f(x)$ whose graph is given below, state the value of each quantity if it exists.



(a) $\lim_{x\to -3^-} f(x) =$ ∞

(b) $\lim_{x\to -3^+} f(x) =$ $-\infty$

(c) $\lim_{x\to 1} f(x) =$ 3

(d) $f(1) =$ 4

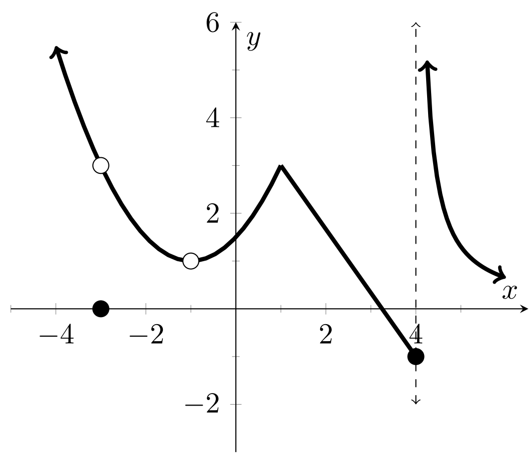
(e) $\lim_{x\to 4^-} f(x) =$ 2

(f) $\lim_{x\to 4^+} f(x) =$ 0

(g) $\lim_{x\to 4} f(x) =$ DNE

(h) $f(4) =$ 2

$\boxed{2}$ (10 points) A graph of the function $f(x)$ is displayed below.



(a) (6 points) From the graph of f, state the numbers at which f is discontinuous and why.

At $x = -3$ as $\lim_{x\to -3} f(x) \neq f(-3)$

At $x = -1$ as $f(-1)$ is undefined / DNE

At $x=4$ as $\lim_{x\to 4} f(x)$ DNE

(b) (4 points) From the graph of f, state the numbers at which f fails to be differentiable and why.

At $x = -3, -1, 4$ as f is not continuous.

At $x = 1$ as f has a corner / cusp.

3 (18 points) Evaluate the following limits. Justify your answers with words and/ or any relevent algebra. Be sure to use proper notation, as points will be deducted for not doing so.

(a) $\displaystyle\lim_{x\to-5}\frac{x^2+5x}{x^2+6x+5} = \lim_{x\to-5}\frac{x(x+5)}{(x+5)(x+1)}$

$\displaystyle = \lim_{x\to-5}\frac{x}{x+1}$

$\displaystyle = \frac{-5}{-6}$

$\displaystyle = \boxed{\frac{5}{6}}$

(b) $\displaystyle\lim_{x\to2}\ln\left(\frac{8-x^2}{1+x}\right) = \ln\left(\lim_{x\to2}\frac{8-x^2}{1+x}\right)$

$\displaystyle = \ln\left(\frac{8-4}{1+2}\right)$

$\displaystyle = \boxed{\ln(4/3)}$

(c) $\displaystyle\lim_{x\to\infty}\frac{(1-x^3)\,1/x^2}{(x^2-x+1)\,1/x^2} = \lim_{x\to\infty}\frac{1/x^2 - x}{1-1/x+1/x^2}$

$\displaystyle = \frac{-\infty}{1}$

$\displaystyle = \boxed{-\infty}$

4 (18 points) Evaluate the following limits. Justify your answers with words and/ or any relevent algebra. Be sure to use proper notation, as points will be deducted for not doing so.

(a) $\lim\limits_{x\to 9^-} \dfrac{-\sqrt{x}}{(x-9)^3} = \boxed{\infty}$

As $x \to 9^-$, $-\sqrt{x}$ approaches -3, a negative constant.

As $x \to 9^-$ the denominator approaches 0 and is negative.

A negative constant divided by a small negative results in infinity.

(b) $\lim\limits_{x\to 2} \dfrac{\frac{1}{x^2}-\frac{1}{4}}{x-2} = \lim\limits_{x\to 2} \left(\dfrac{4-x^2}{4x^2}\right)\cdot\left(\dfrac{1}{x-2}\right)$

$= \lim\limits_{x\to 2} \dfrac{(2-x)(2+x)}{4x^2(x-2)}$

$= \lim\limits_{x\to 2} \dfrac{-(2+x)}{4x^2}$

$= \dfrac{-4}{4\cdot 4}$

$= \boxed{\dfrac{-1}{4}}$

(c) $\lim\limits_{x\to -\infty} \dfrac{\sqrt{1+9x^6}}{5+x^3} = \lim\limits_{x\to\infty} \dfrac{\sqrt{1+9(-x)^6}}{5+(-x)^3}$

$= \lim\limits_{x\to\infty} \dfrac{\sqrt{1+9x^6}\,(1/x^3)}{(5-x^3)(1/x^3)}$

$= \lim\limits_{x\to\infty} \dfrac{\sqrt{1/x^6+9}}{5/x^3-1}$

$= \dfrac{\sqrt{9}}{-1}$

$= \boxed{-3}$

5 (10 points) Given $f(x) = \begin{cases} 3 & x \geq 2 \\ \frac{3x-6}{|x-2|} & x < 2 \end{cases}$ find $\lim_{x\to 2} f(x)$ or explain why this limit does not exist.

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{3x-6}{|x-2|}$$
$$= \lim_{x\to 2^-} \frac{3(x-2)}{-(x-2)}$$
$$= \lim_{x\to 2^-} (-3)$$
$$= -3.$$
$$\lim_{x\to 2^+} f(x) = 3$$

Since $\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$, $\lim_{x\to 2} f(x)$ does not exist.

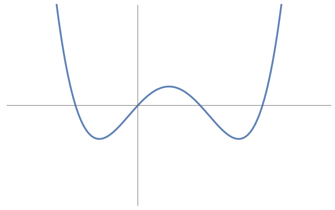
6 (8 points)

Using complete sentences, use the Interemdiate Value Theorem to show that there is a root of the equation $e^x = 5 - 2x$ in the interval $(0, 2)$.

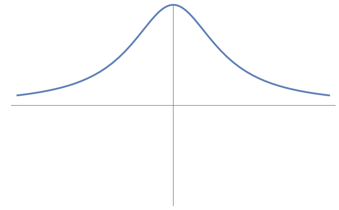
Note, $e^x = 5-2x$ is equivalent to $e^x + 2x - 5 = 0$.
Let $f(x) = e^x + 2x - 5$. Note that $f(x)$ is a continuous function on $(0,2)$.
Also $f(0) = e^0 + 2(0) - 5 = -4 < 0$ and
$f(2) = e^2 + 4 - 5 = e^2 - 1 > 0$.

Since $f(0) < 0$ and $f(2) > 0$ we know that $f(c) = 0$ for some c in $(0,2)$.
Thus the given equation has a root in $(0,2)$.

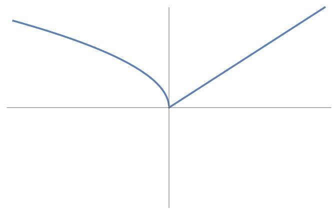
$\boxed{7}$ (8 points) Match the graph of each function (a) - (d) with the graph of its derivative I-VIII. Please put your answers in the blanks provided below the graphs.



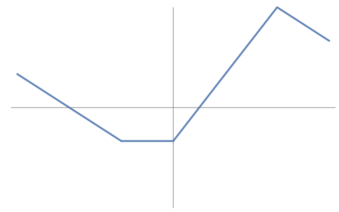
a.)



c.)



b.)



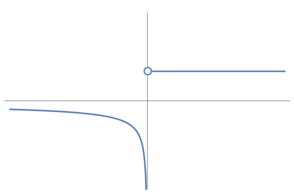
d.)

(a) Graph (a)'s derivative is given by V

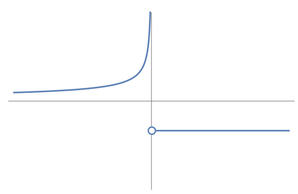
(b) Graph (b)'s derivative is given by I

(c) Graph (c)'s derivative is given by IV

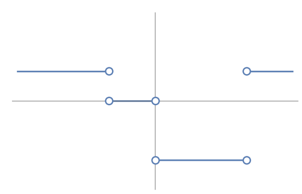
(d) Graph (d)'s derivative is given by VIII



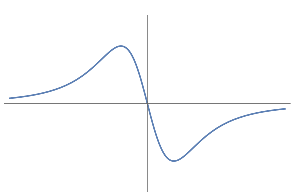
I.



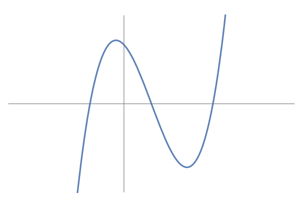
II.



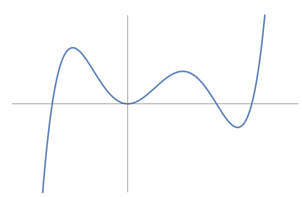
III.



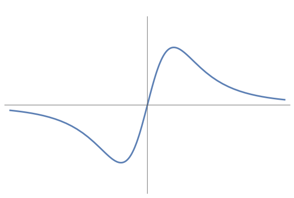
IV.



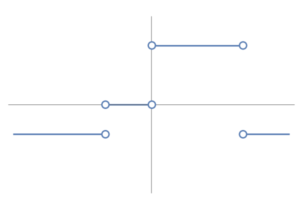
V.



VI.



VII.



VIII.

8 (6 points)

Given $f(x) = \frac{2}{x}$ the derivative of $f(x)$ is given by $f'(x) = -\frac{2}{x^2}$. Using this derivative find the equation of the tangent line to $f(x)$ when $x = 1$. Give your final answer in slope-intercept form.

Point: $f(1) = 2/1 = 2$ or $(1,2)$

slope: $m = f'(1) = -2/1 = -2$

$y - y_1 = m(x - x_1)$

$y - 2 = -2(x-1)$

$y - 2 = -2x + 2$

$\boxed{y = -2x + 4}$

9 (10 points)

(a) (2 points) State the limit definition of the derivative of the function $f(x)$.

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

(b) (8 points)
Given $f(x) = \sqrt{5x}$, find $f'(x)$ using the definition. **No credit will be given for answers found using derivative short-cut formulas**

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{5(x+h)} - \sqrt{5x}}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{5x+5h} - \sqrt{5x})}{h} \frac{(\sqrt{5x+5h} + \sqrt{5x})}{(\sqrt{5x+5h} + \sqrt{5x})}$$

$$= \lim_{h \to 0} \frac{5x + 5h - 5x}{h(\sqrt{5x+5h} + \sqrt{5x})}$$

$$= \lim_{h \to 0} \frac{5h}{h(\sqrt{5x+5h} + \sqrt{5x})}$$

$$= \lim_{h \to 0} \frac{5}{\sqrt{5x+5h} + \sqrt{5x}}$$

$$= \frac{5}{\sqrt{5x} + \sqrt{5x}}$$

$$= \boxed{\frac{5}{2\sqrt{5x}}}$$

$\boxed{10}$ (4 points) The number of bacteria after t hours in a controlled laboratory setting is given by the function $n = f(t)$ where n is the number of bacteria and t is measured in hours.

(a) Suppose $f'(2) = 100$. What are the units of the derivative?

The units are number of bacteria /hour or number of bacteria per hour.

(b) In the context of this situation, explain what $f'(2) = 100$ means using complete sentences.

At $t=2$ hours the population of bacteria is growing at a rate of 100 bacteria per hour.

$\boxed{11}$ (5 points)

Prove that $\lim_{x\to 0} x^2 \cos\frac{4}{x} = 0$. You must clearly explain your work and cite any relevent theorems for full credit.

Note $-1 \leq \cos\frac{4}{x} \leq 1$.

As x^2 is positive, $-x^2 \leq x^2\cos\left(\frac{4}{x}\right) \leq x^2$.

Notice $\lim_{x\to 0}(-x^2) = 0$ and $\lim_{x\to 0} x^2 = 0$.

Thus $\lim_{x\to 0} x^2\cos\left(\frac{4}{x}\right) = 0$ by the Squeeze theorem.