

Your Name

Your Signature

Instructor Name

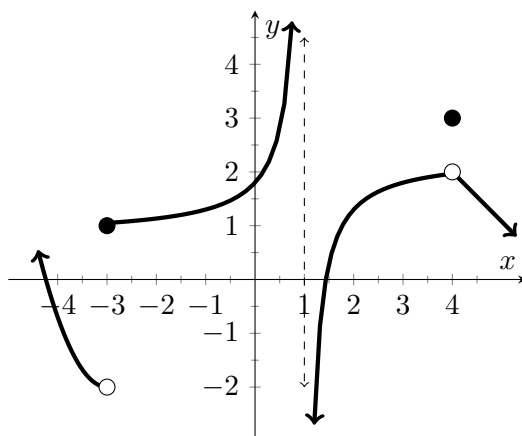
End Time

| Problem | Total Points | Score |
|--------------|--------------|-------|
| 1 | 8 | |
| 2 | 10 | |
| 3 | 18 | |
| 4 | 18 | |
| 5 | 10 | |
| 6 | 8 | |
| 7 | 8 | |
| 8 | 6 | |
| 9 | 10 | |
| 10 | 4 | |
| Extra Credit | (5) | |
| Total | 100 | |

- This test is closed notes and closed book.
- You may **not** use a calculator.
- In order to receive full credit, you must **show your work**. Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- **PLACE A BOX AROUND YOUR FINAL ANSWER** to each question where appropriate.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

1 (8 points)

For the function $f(x)$ whose graph is given below, state the value of each quantity if it exists.



(a) $\lim_{x \rightarrow -3^-} f(x) = \underline{\hspace{2cm}}$

(d) $f(-3) = \underline{\hspace{2cm}}$

(g) $\lim_{x \rightarrow 4} f(x) = \underline{\hspace{2cm}}$

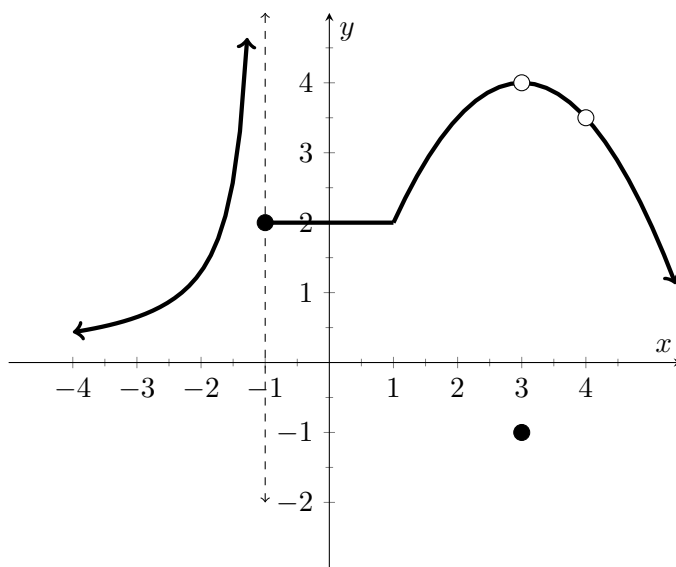
(b) $\lim_{x \rightarrow -3^+} f(x) = \underline{\hspace{2cm}}$

(e) $\lim_{x \rightarrow 1^-} f(x) = \underline{\hspace{2cm}}$

(h) $f(4) = \underline{\hspace{2cm}}$

(c) $\lim_{x \rightarrow -3} f(x) = \underline{\hspace{2cm}}$

(f) $\lim_{x \rightarrow 1^+} f(x) = \underline{\hspace{2cm}}$

2 (10 points) A graph of the function $f(x)$ is displayed below.

(a) (6 points) From the graph of f , state the numbers at which f is discontinuous and why.

(b) (4 points) From the graph of f , state the numbers at which f fails to be differentiable and why.

- 3 (18 points) Evaluate the following limits. Justify your answers with words and/ or any relevant algebra. Be sure to use proper notation, as points will be deducted for not doing so.

(a) $\lim_{x \rightarrow -2} \frac{x^2 + 2x}{x^2 - 3x - 10}$

(b) $\lim_{x \rightarrow 1} \ln \left(\frac{7 - x^2}{1 + x} \right)$

(c) $\lim_{x \rightarrow \infty} \frac{1 + x - 2x^2}{x^3 + 1}$

- 4 (18 points) Evaluate the following limits. Justify your answers with words and/ or any relevant algebra. Be sure to use proper notation, as points will be deducted for not doing so.

(a) $\lim_{x \rightarrow 4^+} \frac{-\sqrt{x}}{(4-x)^3}$

(b) $\lim_{x \rightarrow 4} \frac{\frac{1}{x^2} - \frac{1}{16}}{x - 4}$

(c) $\lim_{x \rightarrow -\infty} \frac{\sqrt{7 + 25x^6}}{2 + x^3}$

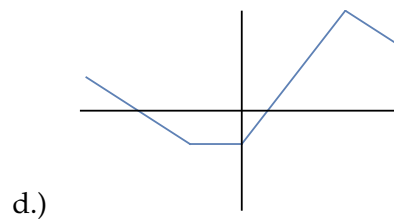
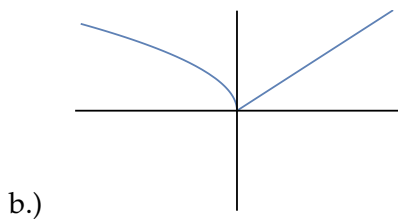
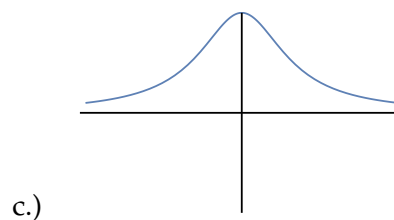
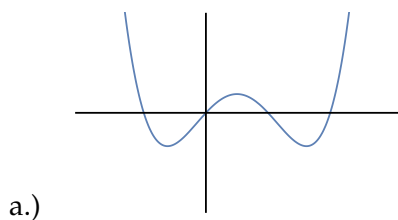
5 (10 points)

Given $f(x) = \begin{cases} 3 & x \geq 3 \\ \frac{3x-9}{|x-3|} & x < 3 \end{cases}$ find $\lim_{x \rightarrow 3} f(x)$ or explain why this limit does not exist.

6 (8 points)

Using complete sentences, use the Intermediate Value Theorem to show that there is a root of the equation $2e^x = 1 + 8x$ in the interval $(0, 1)$.

- 7 (8 points) Match the graph of each function (a) - (d) with the graph of its derivative I-VIII. Please put your answers in the blanks provided below the graphs.

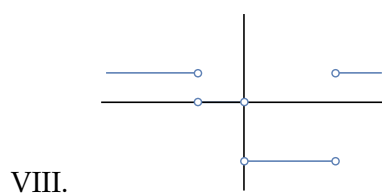
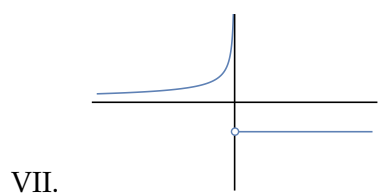
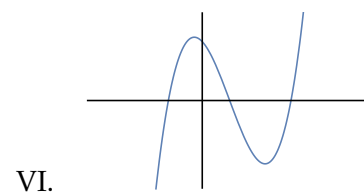
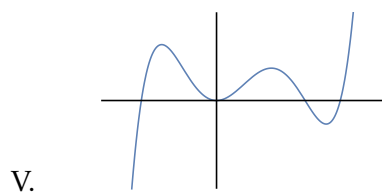
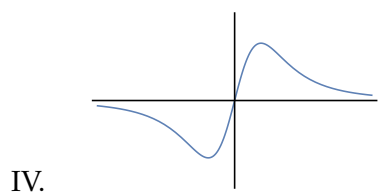
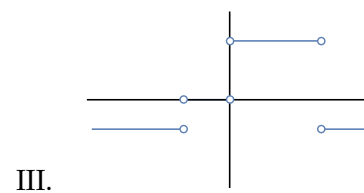
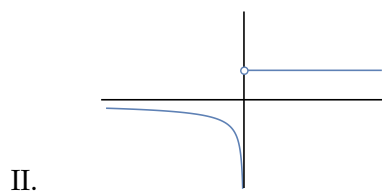
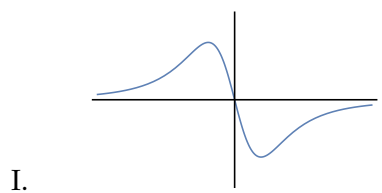


(a) Graph (a)'s derivative is given by _____

(b) Graph (b)'s derivative is given by _____

(c) Graph (c)'s derivative is given by _____

(d) Graph (d)'s derivative is given by _____



8

 (6 points)

Given $f(x) = \frac{8}{x^2}$ the derivative of $f(x)$ is given by $f'(x) = -\frac{16}{x^3}$. Using this derivative find the equation of the tangent line to $f(x)$ when $x = 2$. Give your final answer in slope-intercept form.

9

 (10 points)

(a) (2 points) State the limit definition of the derivative of the function $f(x)$.

(b) (8 points)

Given $f(x) = \sqrt{7x}$, find $f'(x)$ using the definition. **No credit will be given for answers found using derivative short-cut formulas.** Simplify your derivative.

10 (4 points) The number of bacteria after t hours in a controlled laboratory setting is given by the function $n = f(t)$ where n is the number of bacteria and t is measured in hours.

(a) Suppose $f'(10) = -300$. What are the units of the derivative?

(b) In the context of this situation, explain what $f'(10) = -300$ means using complete sentences.

11 (5 points) **Extra Credit** Prove that $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 0$. You must clearly explain your work and cite any relevant theorems for full credit.