Your Name

Your Signature

Instructor Name

End Time

Problem	Total Points	Score
1	8	
2	8	
3	8	
4	8	
5	16	
6	10	
7	10	
8	15	
9	10	
10	7	
Total	100	

- The total time allowed for this exam is 60 minutes.
- This test is closed notes and closed book.
- You may **not** use a calculator.
- In order to receive full credit, you must **show your work.** Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- **PLACE A BOX AROUND** $\boxed{\text{YOUR FINAL ANSWER}}$ **to each question** where appropriate.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

1 (8 points) Find dy/dx when $x^2 + 2xy - y^2 = 7$.

$$2x + 2y + 2xy' - 2yy' = 0$$

$$2xy' - 2yy' = -2x - 2y$$

$$y'(2x - 2y) = -2x - 2y$$

$$y' = \frac{-2x - 2y}{2x - 2y}$$

$$\boxed{y' = \frac{-x - y}{x - y} = \frac{x + y}{y - x}}$$

2 (8 points) Given $y = (\sin x)^x$ find y'.

$$\ln y = \ln(\sin x)^x$$

$$\ln y = x \ln(\sin x)$$

$$\frac{1}{y} y' = \ln(\sin x) + x \cdot \frac{1}{\sin x} \cos x$$

$$y' = \left(\ln(\sin x) + \frac{x \cos x}{\sin x}\right) y$$

$$\boxed{y' = (\ln(\sin x) + x \cot x)(\sin x)^x}$$

$$\boxed{y' = \left(\ln(\sin x) + \frac{x \cos x}{\sin x}\right)(\sin x)^x}$$

3 (8 points)

(a) Find the linearization of $f(x) = \sqrt{5+x^2}$ at $a = 2$.

$f'(x) = \frac{1}{2}(5+x^2)^{-1/2} \cdot 2x$

$= \frac{x}{\sqrt{5+x^2}}$

$f'(2) = 2/3$

$f(2) = \sqrt{9} = 3$

$L(x) = f'(a)(x-a) + f(a)$

$= \frac{2}{3}(x-2) + 3$

$= \frac{2}{3}x - \frac{4}{3} + \frac{9}{3}$

$= \boxed{\frac{2}{3}x + \frac{5}{3}}$

(b) Use linear approximation to estimate the value of $f(x)$ at $a = 2.1$

$f(2.1) \approx L(2.1)$

$= \frac{2}{3}(2.1) + \frac{5}{3}$

$= 2(0.7) + 1.6666...$

$\approx 1.4 + 1.667$

$= \boxed{3.067}$

4 (8 points) Find the absolute maximum and minimum of the function $f(x) = \frac{1}{3}x^3 - 4x^2 + 12x + 1$ on the interval $0 \leq x \leq 3$.

$f'(x) = \frac{1}{3} \cdot 3x^2 - 8x + 12$

$= x^2 - 8x + 12$

$0 = (x-2)(x-6)$

$x = 2, \quad x = 6$ (not in $[0,3]$)

Check: $\boxed{f(0) = 1 \quad \text{absolute min}}$

$f(2) = 8/3 - 16 + 24 + 1$

$= 8/3 + 9(3/3)$

$= 8/3 + 27/3$

$= 35/3 = \boxed{11\tfrac{2}{3} \text{ absolute max}}$

$f(3) = \frac{1}{3} \cdot 27 - 36 + 36 + 1$

$= 9 + 1$

$= 10$

5 (16 points) Evaluate the following limits.

(a) $\lim_{x \to 0} \frac{x - \sin x}{x^2}$ $\left(\frac{0}{0} \text{ form}\right)$

$\overset{H}{=} \lim_{x \to 0} \frac{1 - \cos x}{2x}$ $\left(\frac{0}{0} \text{ form}\right)$

$\overset{H}{=} \lim_{x \to 0} \frac{+\sin x}{2}$

$= \frac{0}{2}$

$= \boxed{0}$ ✓

(b) $\lim_{t \to 0} \frac{t^2 + 5}{\cos t} = \frac{5}{1}$

$= \boxed{5}$ ← not 0/0 so no l'Hospital's rule needed.

(c) $\lim_{x \to \infty} (x^2)^{1/x} = \lim_{x \to \infty} x^{2/x}$ (form ∞^0)

let $y = x^{2/x}$

$\ln y = \ln x^{2/x}$

$= \frac{2}{x} \ln x$

$= \frac{2 \ln x}{x}$

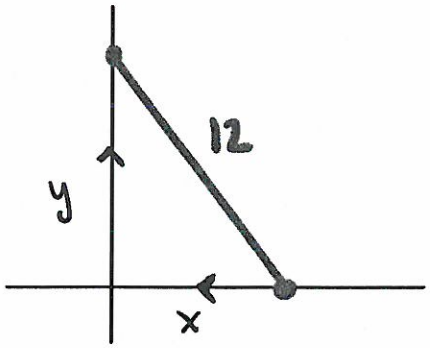
$\lim_{x \to \infty} \frac{2 \ln x}{x} \overset{H}{=} \lim_{x \to 0} \frac{2/x}{1} = 0$

thus $\lim_{x \to \infty} \ln y = 0$

so $\boxed{\lim_{x \to \infty} y = e^0 = 1}$

6 (10 points) A 12 foot ladder is resting against the wall. The bottom is initially 10 feet away from the wall and is being pushed towards the wall at a rate of 1 ft/sec.

(a) Sketch and label a diagram modeling the situation described above.



x starts at 10 feet.

(b) How fast is the top of the ladder moving up the wall 3 seconds after we start pushing? Give your answer using appropriate units.

note $dx/dt = -1$

want dy/dt when $t = 3$ sec.

after 3 sec, $x = 7 \Rightarrow 7^2 + y^2 = 12^2$

$\Rightarrow 49 + y^2 = 144$

$\Rightarrow y^2 = 95$

$\Rightarrow y = \sqrt{95}$

$$x^2 + y^2 = 12^2$$

$$2x\, dx/dt + 2y\, dy/dt = 0$$

$$\cancel{3}\,7(-1) + \sqrt{95}\, dy/dt = 0$$

$$\sqrt{95}\, dy/dt = \cancel{3}\ 7$$

$$\boxed{dy/dt = \cancel{3}\,7/\sqrt{95}\ \text{ft/sec}}$$

$\boxed{7}$ (10 points) Sketch the graph of a function $f(x)$ that satisfies all of the given conditions.

(a) The domain of $f(x)$ is $(-\infty, \infty)$.

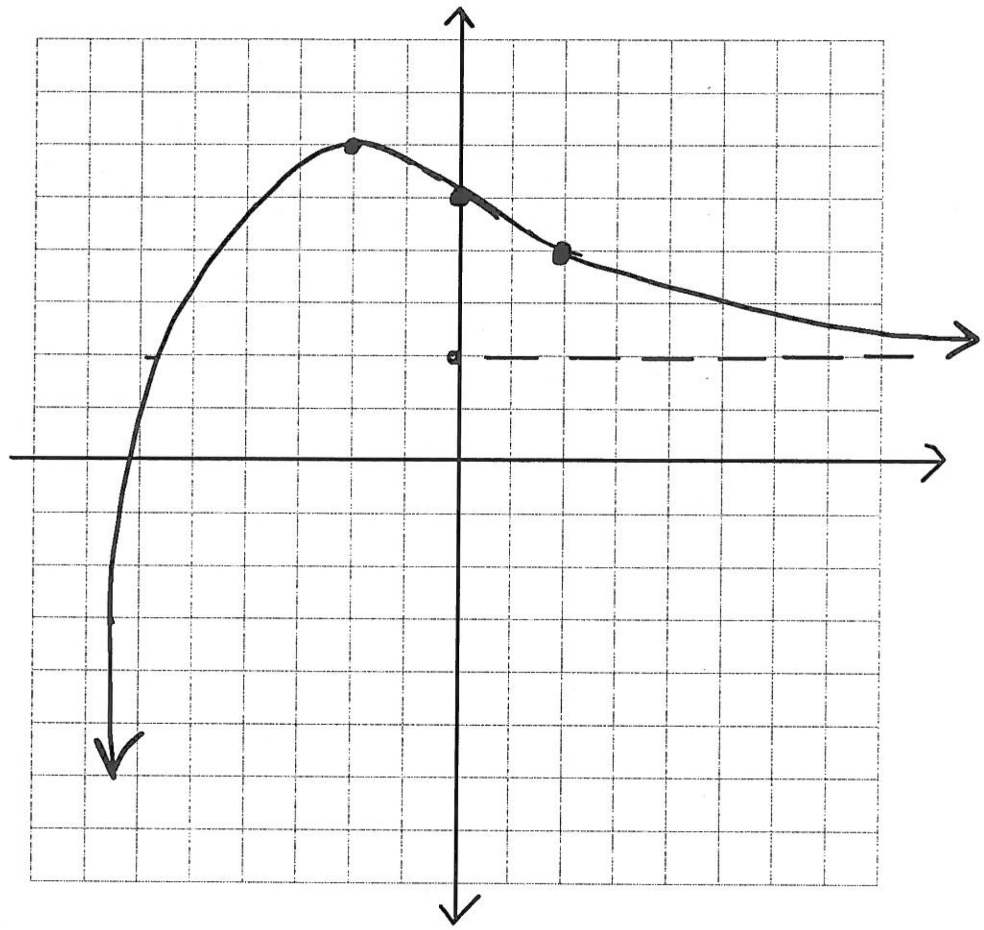
(b) $f(0) = 5$

(c) $\lim_{x \to \infty} f(x) = 2$

increase decrease

(d) $f'(x) > 0$ on the interval $(-\infty, -1)$; $f'(x) < 0$ on the interval $(-1, \infty)$

(e) $f''(x) < 0$ on the interval $(-\infty, 2)$; $f''(x) > 0$ on the interval $(2, \infty)$



8 (15 points) Use the information below to answer questions about the function $f(x)$. Make sure you answer the question!

+4

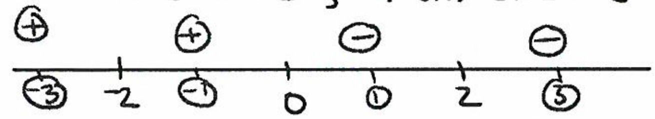
$$f(x) = \frac{x^2}{x^2-4}, \quad f'(x) = \frac{-8x}{(x^2-4)^2}, \quad f''(x) = \frac{8(3x^2+4)}{(x^2-4)^3}.$$

(a) Find the domain.

$x \neq \pm 2$ or $(-\infty,-2) \cup (-2,2) \cup (2,\infty)$

(b) Determine the intervals on which the function is increasing/decreasing.

$f'(x)=0$ @ $x=0$, $f'(x)$ undef @ $x=\pm 2$



f is inc on $(-\infty,-2) \cup (-2,0)$ f is dec on $(0,2) \cup (2,\infty)$

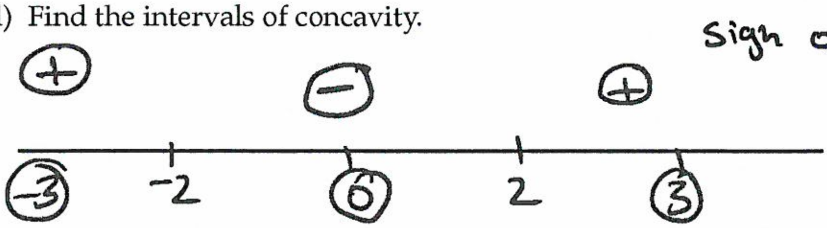
(c) Find the local maximum/minimum values of the function. If something doesn't exist, you must explicitly state this and justify your answer.

$f(0)=4$ is a local max

there is no local min as f(x) is undefined at the other critical numbers

(d) Find the intervals of concavity.

Sign of f''



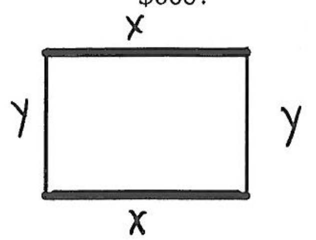
f concave up on $(-\infty,-2) \cup (2,\infty)$
f concave down on $(-2,2)$

(e) Find the inflection points. If there aren't any, you must explicitly state this and justify your answer.

There are no inflection points as f(-2) and f(2) are undefined.

9 (10 points) A rectangular plot of land is to be fenced in using two kinds of fencing. Two opposite sides will use heavy-duty fencing selling for \$3 a foot, while the remaining two sides will use standard fencing selling for \$2 a foot.

(a) What are the dimensions of the rectangular plot of greatest area that can be fenced in at a cost of \$600?



let x = heavy duty
y = normal

$C = 3\cdot 2x + 2\cdot 2y$

$600 = 6x + 4y$

$600 - 6x = 4y$

$y = \frac{600-6x}{4}$

$y = \frac{300-3x}{2}$

$y = 150 - \frac{3}{2}x$

$A = xy$

$= x(150 - 3/2\,x)$

$= 150x - 3/2\,x^2$

$A' = 150 - 3x$

$0 = 150 - 3x$

$3x = 150$

$\boxed{x = 50 \text{ ft on heavy duty}}$

$y = 150 - \frac{3}{2}\cdot 50$

$y = 150 - 75$

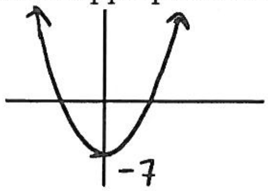
$\boxed{y = 75 \text{ ft on regular}}$

(b) Use the First or Second Derivative Test to justify your conclusion in Part (a).

Note $A''(x) = -3 < 0$. As $A''(x)$ is negative, A is concave down and we have a maximum at $x = 50$, by the second derivative test.

10 (7 points) In this problem we are going to use Newton's method to estimate $\sqrt{7}$ using the function $f(x) = x^2 - 7$.

(a) State an appropriate initial value x_1 for use in applying Newton's method. Justify your answer.

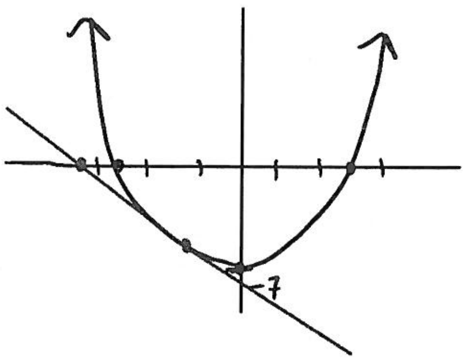


$f(2) = -3$

$f(3) = 2$

there is a zero between 2 & 3, let $\boxed{x_1 = 2.5}$

(b) Sketch the function and illustrate the idea behind Newton's Method using starting point $x_1 = -1$.



(c) Suppose you are given an initial value of $x_1 = -1$. Find the next estimate x_2 given by Newton's method for a root of the function $f(x)$.

$f(x) = x^2 - 7$

$f'(x) = 2x$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= -1 - \frac{(1-7)}{-2}$$

$$= -1 + (-6/2)$$

$$= \boxed{-4}$$

10 (7 points) In this problem we are going to use Newton's method to estimate $\sqrt{7}$ using the function $f(x) = x^2 - 7$.

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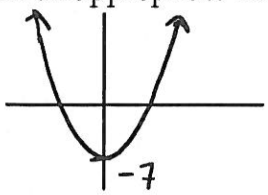

$f(2) = -3$
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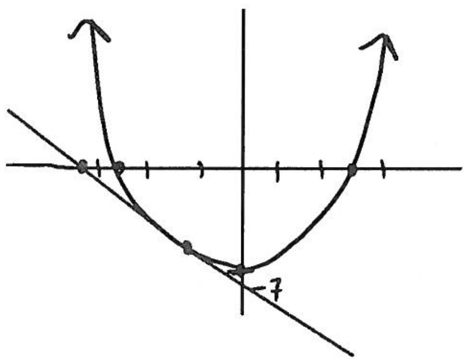

(c) Suppose you are given an initial value of $x_1 = -1$. Find the next estimate x_2 given by Newton's method for a root of the function $f(x)$.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$f(x) = x^2 - 7$
$f'(x) = 2x$

$$= -1 - \frac{(1-7)}{-2}$$

$$= -1 + (-6/2)$$

$$= \boxed{-4}$$