

Your Name

Your Signature

Instructor Name

End Time

Problem	Total Points	Score
1	8	
2	8	
3	8	
4	8	
5	16	
6	10	
7	10	
8	15	
9	10	
10	7	
Total	100	

- The total time allowed for this exam is 60 minutes.
- This test is closed notes and closed book.
- You may **not** use a calculator.
- In order to receive full credit, you must **show your work**. Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- **PLACE A BOX AROUND** YOUR FINAL ANSWER **to each question** where appropriate.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

1 (8 points) Find dy/dx when $x^2 + 2xy - y^2 = 7$.

2 (8 points) Given $y = (\sin x)^x$ find y' .

3 (8 points)

(a) Find the linearization of $f(x) = \sqrt{5 + x^2}$ at $a = 2$.

(b) Use linear approximation to estimate the value of $f(x)$ at $a = 2.1$

4 (8 points) Find the absolute maximum and minimum of the function $f(x) = \frac{1}{3}x^3 - 4x^2 + 12x + 1$ on the interval $0 \leq x \leq 3$.

5 (16 points) Evaluate the following limits.

(a) $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^2}$

(b) $\lim_{t \rightarrow 0} \frac{t^2 + 5}{\cos t}$

(c) $\lim_{x \rightarrow \infty} (x^2)^{1/x}$

- 6 (10 points) A 12 foot ladder is resting against the wall. The bottom is initially 10 feet away from the wall and is being pushed towards the wall at a rate of 1 ft/sec.

- (a) Sketch and label a diagram modeling the situation described above.
- (b) How fast is the top of the ladder moving up the wall 3 seconds after we start pushing? Give your answer using appropriate units.

7 (10 points) Sketch the graph of a function $f(x)$ that satisfies all of the given conditions.

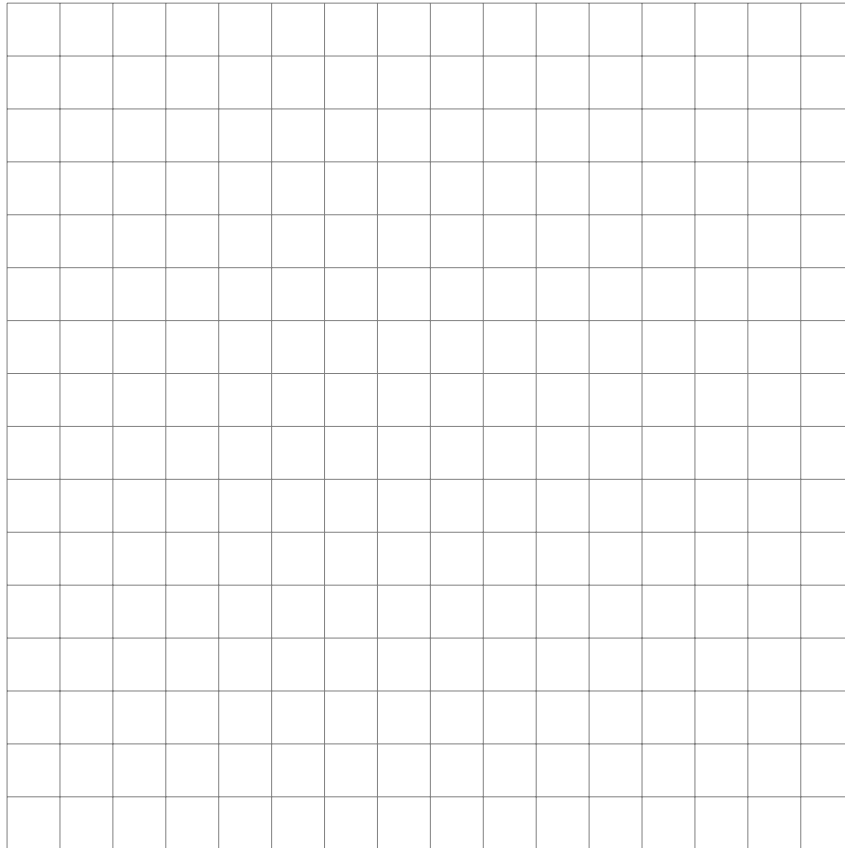
(a) The domain of $f(x)$ is $(-\infty, \infty)$.

(b) $f(0) = 5$

(c) $\lim_{x \rightarrow \infty} f(x) = 2$

(d) $f'(x) > 0$ on the interval $(-\infty, -1)$; $f'(x) < 0$ on the interval $(-1, \infty)$

(e) $f''(x) < 0$ on the interval $(-\infty, 2)$; $f''(x) > 0$ on the interval $(2, \infty)$



- 8 (15 points) Use the information below to answer questions about the function $f(x)$. Make sure you answer the question!

$$f(x) = \frac{x^2}{x^2 - 4} + 4, \quad f'(x) = \frac{-8x}{(x^2 - 4)^2}, \quad f''(x) = \frac{8(3x^2 + 4)}{(x^2 - 4)^3}.$$

- (a) Find the domain.
- (b) Determine the intervals on which the function is increasing/decreasing.
- (c) Find the local maximum/minimum values of the function. If something doesn't exist, you must explicitly state this and justify your answer.
- (d) Find the intervals of concavity.
- (e) Find the inflection points. If there aren't any, you must explicitly state this and justify your answer.

9 (10 points) A rectangular plot of land is to be fenced in using two kinds of fencing. Two opposite sides will use heavy-duty fencing selling for \$3 a foot, while the remaining two sides will use standard fencing selling for \$2 a foot.

(a) What are the dimensions of the rectangular plot of greatest area that can be fenced in at a cost of \$600?

(b) Use the First or Second Derivative Test to justify your conclusion in Part (a).

10 (7 points) In this problem we are going to use Newton's method to estimate $\sqrt{7}$ using the function $f(x) = x^2 - 7$.

- (a) State an appropriate initial value x_1 for use in applying Newton's method. Justify your answer.
- (b) Sketch the function and illustrate the idea behind Newton's Method using starting point $x_1 = -1$.
- (c) Suppose you are given an initial value of $x_1 = -1$. Find the next estimate x_2 given by Newton's method for a root of the function $f(x)$.