Your Name

Your Signature

Instructor Name

End Time

Problem	Total Points	Score
1	8	
2	8	
3	8	
4	8	
5	16	
6	10	
7	10	
8	15	
9	10	
10	7	
Total	100	

- The total time allowed for this exam is 60 minutes.
- This test is closed notes and closed book.
- You may **not** use a calculator.
- In order to receive full credit, you must **show your work.** Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- **PLACE A BOX AROUND YOUR FINAL ANSWER to each question** where appropriate.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

1 (8 points) Find dy/dx when $x^2 + 3xy - y^2 = -5$.

$$2x + 3y + 3xy' - 2yy' = 0$$

$$3xy' - 2yy' = -2x - 3y$$

$$y'(3x - 2y) = -2x - 3y$$

$$\boxed{y' = \frac{-2x - 3y}{3x - 2y}}$$

$$\boxed{y' = \frac{2x + 3y}{2y - 3x}}$$

2 (8 points) Given $y = (\cos x)^x$ find y'.

$$\ln y = \ln (\cos x)^x$$

$$\ln y = x \ln(\cos x)$$

$$\frac{1}{y} y' = \ln(\cos x) + x \cdot \frac{1}{\cos x}(-\sin x)$$

$$y' = (\ln(\cos x) - x \tan x)\, y$$

$$\boxed{y' = (\ln(\cos x) - x \tan x)(\cos x)^x}$$

3 (8 points)

(a) Find the linearization of $f(x) = \sqrt{9+x^2}$ at $a = 4$.

$$f'(x) = \frac{1}{2}(9+x^2)^{-1/2} \cdot 2x$$
$$= \frac{x}{\sqrt{9+x^2}}$$
$$f'(4) = 4/\sqrt{25} = 4/5$$
$$f(4) = \sqrt{25} = 5$$

$$L(x) = f(a) + f'(a)(x-a)$$
$$= 5 + 4/5\,(x-4)$$
$$= 5 + 4/5\,x - 16/5$$
$$= \boxed{4/5\,x + 9/5}$$

(b) Use linear approximation to estimate the value of $f(x)$ at $a = 4.1$.

$$f(4.1) \approx L(4.1)$$
$$= 4/5\,(4.1) + 9/5$$
$$= 0.8\,(4.1) + 1.8$$
$$= 3.28 + 1.8$$
$$= \boxed{5.08}$$

$$\begin{array}{r} 41 \\ \times\ 8 \\ \hline 328 \end{array}$$

4 (8 points) Find the absolute maximum and minimum of the function $f(x) = \frac{1}{3}x^3 - 3x^2 + 8x - 1$ on the interval $0 \le x \le 3$.

$$f'(x) = x^2 - 6x + 8$$
$$0 = (x-2)(x-4)$$

$x = 2$ is in $[0,3]$

Test critical # + endpoints

$$f(0) = -1$$

$$f(2) = 8/3 - 12 + 16 - 1$$
$$= 8/3 + 3$$
$$= 17/3 = 5\tfrac{2}{3}$$

$$f(3) = 1/3(27) - 27 + 24 - 1$$
$$= 9 - 4$$
$$= 5$$

absolute min $f(0) = -1$
absolute max $f(2) = 17/3$

5 (16 points) Evaluate the following limits.

(a) $\lim_{x\to 0} \frac{x^2}{1-\cos x} \overset{H}{=} \lim_{x\to 0} \frac{2x}{\sin x} \quad \left(\frac{0}{0}\right)$

$\overset{H}{=} \lim_{x\to 0} \frac{2}{\cos x} \quad \left(\frac{0}{0}\right)$

$= \frac{2}{1}$

$= \boxed{2}$

$\lim_{t\to 0} \frac{t^2+4}{\cos t} = \frac{4}{\cos 0} = \boxed{4}$

(b) $\lim_{x\to\infty} (x^2)^{1/x} = \lim_{x\to\infty} x^{2/x}$

let $y = x^{2/x}$, $\ln y = \frac{2}{x} \ln x$

$\lim_{x\to\infty} \frac{2\ln x}{x} = \lim_{x\to\infty} \frac{2/x}{1}$

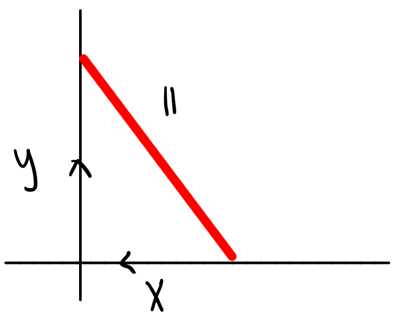
$= 0$

thus $\lim_{x\to\infty} \ln y = 0$ and $\boxed{\lim_{x\to\infty} y = e^0 = 1}$

6 (10 points)

An 11 foot ladder is resting against the wall. The bottom is initially 9 feet away from the wall and is being pushed towards the wall at a rate of 1 ft/sec.

(a) Sketch and label a diagram modeling the situation described above.



(b) How fast is the top of the ladder moving up the wall 2 seconds after we start pushing? Give your answer using appropriate units.

Know $dx/dt = -1$

Want dy/dt when 2 sec have past or $9-4=5$ ft $= x$

$x^2 + y^2 = 121$

$5^2 + y^2 = 121$

$25 + y^2 = 121$

$y^2 = 96$

$y = \sqrt{96}$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$5(-1) + \sqrt{96}\, dy/dt = 0$

$\sqrt{96}\, dy/dt = 5$

$\boxed{dy/dt = 5/\sqrt{96} \text{ ft/sec}}$

$\boxed{7}$ (10 points) Sketch the graph of a function $f(x)$ that satisfies all of the given conditions.

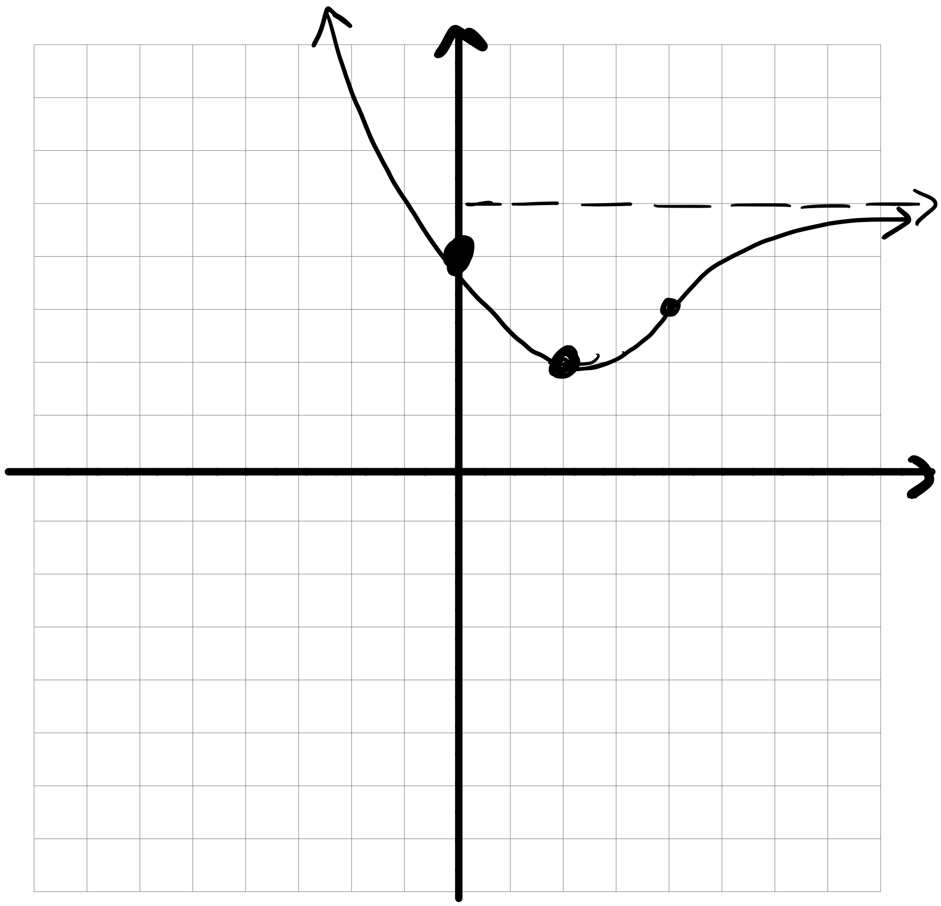
(a) The domain of $f(x)$ is $(-\infty, \infty)$.

(b) $f(0) = 4$

(c) $\lim_{x\to\infty} f(x) = 5$

(d) $f'(x) < 0$ on the interval $(-\infty, 1)$; $f'(x) > 0$ on the interval $(1, \infty)$

(e) $f''(x) > 0$ on the interval $(-\infty, 3)$; $f''(x) > 0$ on the interval $(3, \infty)$



8 (15 points) Use the information below to answer questions about the function $f(x)$. Make sure you answer the question!

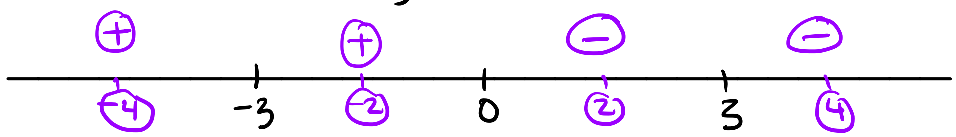
$$f(x) = \frac{x^2}{x^2-9} + 2, \qquad f'(x) = \frac{-18x}{(x^2-9)^2}, \qquad f''(x) = \frac{54(x^2+3)}{(x^2-9)^3}.$$

(a) Find the domain.

$x \neq \pm 3$ or $\boxed{(-\infty,-3)\cup(-3,3)\cup(3,\infty)}$

(b) Determine the intervals on which the function is increasing/decreasing.

$f'(x) = 0$ @ $x=0$; $f'(x)$ undef @ ± 3



$\boxed{f \text{ is inc on } (-\infty,-3)\cup(-3,0) \quad f \text{ is dec on } (0,3)\cup(3,\infty)}$

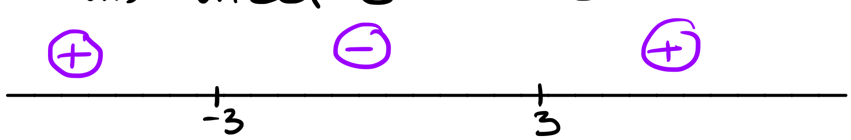
(c) Find the local maximum/minimum values of the function. If something doesn't exist, you must explicitly state this and justify your answer.

$f(0) = 2$ is a local max

there is no local min as f is undefined at $x = \pm 3$

(d) Find the intervals of concavity.

$f''(x)$ undef @ $x = \pm 3$



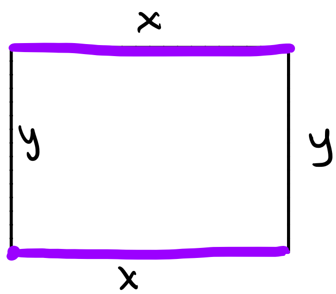
$\boxed{f \text{ concave up on } (-\infty,-3)\cup(3,\infty);\ f \text{ concave down } (-3,3)}$

(e) Find the inflection points. If there aren't any, you must explicitly state this and justify your answer.

There are no inflection points as the only places f'' changes sign occur where f'' is undefined.

9 (10 points) A rectangular plot of land is to be fenced in using two kinds of fencing. Two opposite sides will use heavy-duty fencing selling for \$3 a foot, while the remaining two sides will use standard fencing selling for \$2 a foot.

(a) What are the dimensions of the rectangular plot of greatest area that can be fenced in at a cost of \$1200?



$C = 3 \cdot 2x + 2 \cdot 2y$

$1200 = 6x + 4y$

$4y = 1200 - 6x$

$y = 300 - \frac{3}{2}x$

$A = xy$

$A = x(300 - \frac{3}{2}x)$

$A = 300x - \frac{3}{2}x^2$

$A' = 300 - \frac{9}{2}x$

$0 = 300 - \frac{9}{2}x$

$\frac{9}{2}x = 300$

$x = 300(\frac{2}{9})$

$\boxed{x = \frac{200}{3} \text{ ft}}$

$y = 300 - \frac{3}{2} \cdot \frac{200}{3}$

$= 300 - 100$

$= \boxed{200 \text{ ft}}$

(b) Use the First or Second Derivative Test to justify your conclusion in Part (a).

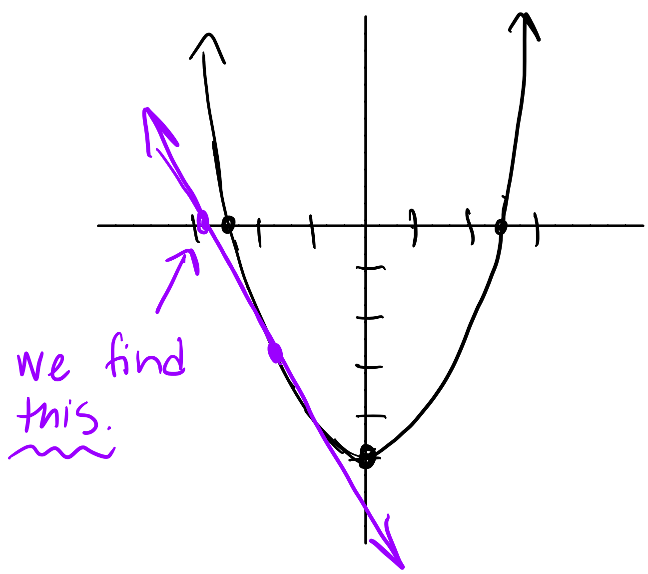
Note $A''(x) = -\frac{9}{2} < 0$. Thus A is concave down & we have a max at $x = \frac{200}{3}$ ft.

10 (7 points) In this problem we are going to use Newton's method to estimate $\sqrt{5}$ using the function $f(x) = x^2 - 5$.

(a) State an appropriate initial value x_1 for use in applying Newton's method. Justify your answer.

We know $f(2)=-1$ and $f(3)=4$, thus there is a zero in $(2,3)$. Choose $x_1=2.5$

(b) Sketch the function and illustrate the idea behind Newton's Method using starting point $x_1 = -1$.



(c) Suppose you are given an initial value of $x_1 = -1$. Find the next estimate x_2 given by Newton's method for a root of the function $f(x)$.

$$f(x) = x^2 - 5$$
$$f'(x) = 2x$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= -1 - \frac{(1-5)}{-2}$$
$$= -1 + \frac{-4}{2}$$
$$= -1 - 2$$
$$= \boxed{-3}$$