

Your Name

Your Signature

Instructor Name

End Time

Problem	Total Points	Score
1	8	
2	8	
3	8	
4	8	
5	16	
6	10	
7	10	
8	15	
9	10	
10	7	
Total	100	

- The total time allowed for this exam is 60 minutes.
- This test is closed notes and closed book.
- You may **not** use a calculator.
- In order to receive full credit, you must **show your work**. Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- **PLACE A BOX AROUND** YOUR FINAL ANSWER **to each question** where appropriate.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

1 (8 points) Find  $dy/dx$  when  $3xy + 2x^2 - y^2 = 1$ .

2 (8 points) Given  $y = x^{\cos x}$  find  $y'$ .

3 (8 points)

(a) Find the linearization of  $f(x) = \sqrt{7 + x^2}$  at  $a = 3$ .

(b) Use linear approximation to estimate the value of  $f(x)$  at  $a = 3.1$ .

4 (8 points) Find the absolute maximum and minimum of the function  $f(x) = \frac{1}{3}x^3 + 2x^2 - 12x + 1$  on the interval  $0 \leq x \leq 3$ .

5 (16 points) Evaluate the following limits.

(a)  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

(b)  $\lim_{t \rightarrow 0} \frac{t^2 + 3}{\cos t}$

(c)  $\lim_{x \rightarrow \infty} (x^2)^{1/x}$

- 6 (10 points) A 9 foot ladder is resting against the wall. The bottom is initially 7 feet away from the wall and is being pushed towards the wall at a rate of 1 ft/sec.

- (a) Sketch and label a diagram modeling the situation described above.
- (b) How fast is the top of the ladder moving up the wall 3 seconds after we start pushing? Give your answer using appropriate units.

7 (10 points) Sketch the graph of a function  $f(x)$  that satisfies all of the given conditions.

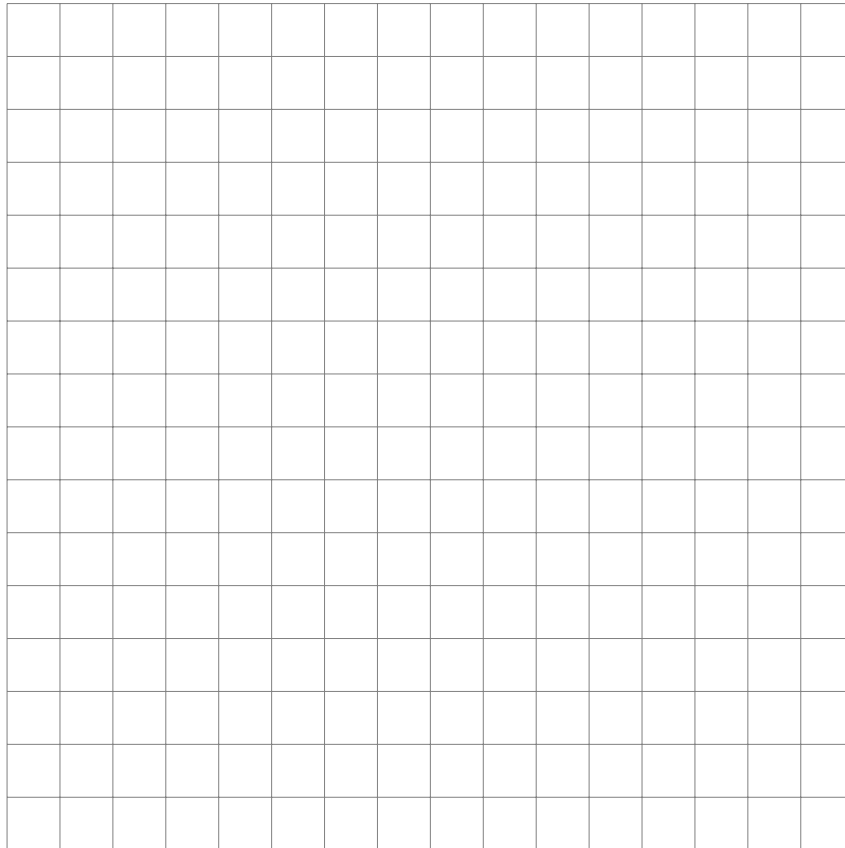
(a) The domain of  $f(x)$  is  $(-\infty, \infty)$ .

(b)  $f(0) = 3$

(c)  $\lim_{x \rightarrow -\infty} f(x) = 1$

(d)  $f'(x) > 0$  on the interval  $(-\infty, 1)$ ;  $f'(x) < 0$  on the interval  $(1, \infty)$

(e)  $f''(x) > 0$  on the interval  $(-\infty, -2)$ ;  $f''(x) < 0$  on the interval  $(-2, \infty)$



- 8 (15 points) Use the information below to answer questions about the function  $f(x)$ . Make sure you answer the question!

$$f(x) = \frac{x^2}{x^2 - 1} + 3, \quad f'(x) = \frac{-2x}{(x^2 - 1)^2}, \quad f''(x) = \frac{2(3x^2 + 1)}{(x^2 - 1)^3}.$$

- (a) Find the domain.
- (b) Determine the intervals on which the function is increasing/decreasing.
- (c) Find the local maximum/minimum values of the function. If something doesn't exist, you must explicitly state this and justify your answer.
- (d) Find the intervals of concavity.
- (e) Find the inflection points. If there aren't any, you must explicitly state this and justify your answer.

9 (10 points) A rectangular plot of land is to be fenced in using two kinds of fencing. Two opposite sides will use heavy-duty fencing selling for \$3 a foot, while the remaining two sides will use standard fencing selling for \$2 a foot.

(a) What are the dimensions of the rectangular plot of greatest area that can be fenced in at a cost of \$360?

(b) Use the First or Second Derivative Test to justify your conclusion in Part (a).



10 (7 points) In this problem we are going to use Newton's method to estimate  $\sqrt{8}$  using the function  $f(x) = x^2 - 8$ .

- (a) State an appropriate initial value  $x_1$  for use in applying Newton's method. Justify your answer.
- (b) Sketch the function and illustrate the idea behind Newton's Method using starting point  $x_1 = -1$ .
- (c) Suppose you are given an initial value of  $x_1 = -1$ . Find the next estimate  $x_2$  given by Newton's method for a root of the function  $f(x)$ .