December 17, 2007 Faudree

NAME:\_\_\_\_\_

There are 8 questions on this exam for a total of 110 points. You may not use calculators, books, or notes. You must show your work to receive full credit. You should simplify numerical answers.

You have two hours to complete the exam.

May the Force be with you.

problem	points
1	
2	
3	
4	
5	
6	
7	
8	
total	

1. (5 points each) Evaluate the limits below. Give the most complete answer. Show your work or explain your reasoning.

(a) 
$$\lim_{x \to 0} \frac{x^4 - 2x^2}{x^3 + x^2}$$

(b) 
$$\lim_{x \to -5^+} \frac{e^x}{x^2 - 25}$$

(c) 
$$\lim_{x \to 0} \frac{e^{2x} - 1}{3x + x^2}$$

(d) 
$$\lim_{x \to \infty} \frac{5 + \sqrt{2x^4 + x}}{7x^2 + x^{2/3}}$$

2. (5 points each) Find dy/dx for each of the following.

(a) 
$$y = e^{2x} \cos(x^3 + e^2)$$

(b) 
$$y = \ln(x + \sec(3x))$$

(c) 
$$xe^y = y - 1$$

3. (5 points each) Evaluate the following integrals.

(a) 
$$\int \frac{7\sin\theta}{(\cos\theta)^2} d\theta$$

(b) 
$$\int_{1}^{2} \frac{e^{1/t}}{t^2} dt$$

(c) 
$$\int_{1/2}^{1} \sin(\pi x) \cos(\pi x) dx$$

- 4. (5 points each) Let  $f(x) = \frac{x^2 + x + 1}{x^2}$ ;  $f'(x) = -\frac{x+2}{x^3}$ ;  $f''(x) = \frac{2(x+3)}{x^4}$ 
  - (a) Identify any asymptotes and show your answer is correct.

(b) Identify the x-value of any local extrema and show your answer is correct.

(c) Identify the x-values of any inflection points and show your answer is correct.

5. (10 points) Find the equation of the line tangent to the curve  $f(x) = \frac{1}{2+2\sin x}$  at x = 0.

6. (10 points) Let R be the region below the graph of  $y = 4 - x^2$  and above the x-axis. Find the dimensions of the rectangle of largest area that can be inscribed in R assuming the base of the rectangle is on the x-axis.

- 7. (5 points each) Let R be the region bounded by  $y = x^2$ ,  $y = \frac{1}{x}$ , and x = 1/2.
  - (a) Sketch the region R.

(b) Set up but do not evaluate an integral for the volume of the solid obtained by rotating the region R about the x-axis.

(c) Set up but do not evaluate an integral for the volume of the solid obtained by rotating the region R about the line x = 3.

- 8. (2 points each) Answer the following.
  - (a) (Complete the following definition.) The derivative of a function f(x) is denoted f'(x) as is defined as:

,

$$f'(x) =$$

provided...

(b) (Complete the following statement.) If  $\frac{dy}{dx} = 35y$  and y(0) = 12, then

$$y =$$

(c) Give an example of a definite integral for which the Fundamental Theorem of Calculus does not apply and explain why it does not apply.

(d) Give an example of a function f(x) and an x-value a such that f is continuous at x = a but not differentiable at x = a.

(e) Assume d(x) measures the density of a wire, in grams per centimeter (g/cm), where x is measured in centimeters (cm) from one end of the wire. Interpret the statement d'(3) = -0.5.