(given:) October 26, 2007

NOTE: There were multiple versions of the exam. Read the problem carefully, it may be slightly different from the one on YOUR midterm.

- 1. Find the derivative of each of the following:
 - (a) $y = \sqrt[3]{x} \frac{10}{x^2} + \ln x + e^2$ Straight applications of the power rule and derivative of natural log. $y' = (1/3)x^{-2/3} + 20x^{-3} + x^{-1}$
 - (b) $g(x) = 5 \sin^{-1}(4x)$. Application of derivative inverse sine and chain rule. $g'(x) = 5(\frac{4}{\sqrt{1-(4x)^2}}) = \frac{20}{\sqrt{1-16x^2}}$
 - (c) $y = e^{-2x} \csc x$ Product rule and chain rule for the exponential function. $y' = -2e^{-2x} \csc x + e^{-2x}(-\csc x \cot x) = -(\csc x)e^{-2x}(2 + \cot x)$
 - (d) $y = (x + \tan(3x))^8$ Application of the chain rule twice over. $y' = 8(x + \tan(3x))^7(1 + 3\sec^2(3x))$
- 2. Find dy/dx for

$$3xe^y = 5 + y^2 + x^3.$$

We will take the derivative implicitly, applying the product rule on the left side. $3e^{y} + 3xe^{y}\frac{dy}{dx} = 2y\frac{dy}{dx} + x^{3}$ $3xe^{y}\frac{dy}{dx} - 2y\frac{dy}{dx} = x^{3} - 3e^{y}$ $(3xe^{y} - 2y)\frac{dy}{dx} = x^{3} - 3e^{y}$ $\frac{dy}{dx} = \frac{x^{3} - 3e^{y}}{(3xe^{y} - 2y)}$

- 3. Find the derivative of the function $y = (x^2 + 1)^{\sin x}$. Logarithmic differentiation is required here. $\ln y = \ln((x^2 + 1)^{\sin x})$ or $\ln y = \sin x \ln(x^2 + 1)$ Now we take the derivative implicitly, using the product rule on the right side. $\frac{1}{y} \frac{dy}{dx} = \cos x \ln(x^2 + 1) + \sin x \frac{2x}{x^2 + 1}$ $\frac{dy}{dx} = y[\cos x \ln(x^2 + 1) + \sin x \frac{2x}{x^2 + 1}]$ $\frac{dy}{dx} = (x^2 + 1)^{\sin x} [\cos x \ln(x^2 + 1) + \sin x \frac{2x}{x^2 + 1}]$
- 4. Use linearization or differentials to approximate $(0.99)^6$. We pick $f(x) = x^6$ and a = 1. Now, we will write the equation of the line tangent to

f at a. Such a line contains the point P = (1, f(1)) = (1, 1) and has slope $f'(1) = 6(1)^5 = 6$. So the line is: y = 1 + 6(x - 1). Now we plug in x = 0.99 into our line to get our approximation: y = 1 + 6(0.99 - 1) = 1 - 0.06 = 0.94.

- 5. The position of a particle is given by the equation $s = 4t^2 16t + 25$ where t is measured in seconds and s is measured in feet.
 - (a) Find the velocity and acceleration of the particle. velocity: v = s' = 8t - 16acceleration: a = v' = s" = 8
 - (b) When is the particle moving to the right? The particle moves to the right when v > 0. So, we set v = 8t - 16 > 0 and solve for t to get: t > 2, the answer.
 - (c) What is the location(s) of the particle when it is at rest? We know the particle is at rest when v = 8t - 16 = 0 or t = 2. This tell us WHEN the particle is at rest. To find WHERE the particle is, we plug t = 2 into the position equation $s(2) = 4(2^2) - 16 * 2 + 25 = 9$ (feet).
- 6. Use the quotient rule to find the derivative of $f(x) = \frac{3x^2 + \sqrt{2}}{\cos x}$. $f'(x) = \frac{6x \cos x - (3x^2 + \sqrt{2})(-\sin x)}{\cos^2(x)} = \frac{6x \cos x + (3x^2 + \sqrt{2}) \sin x}{\cos^2(x)}.$
- 7. The weight, W, in pounds, of a person is a function of the age, a, of the person in years. That is, we have W=f(a).
 - (a) What are the units of f'(a)? pounds per year or lbs/yr
 - (b) What does f'(6) = 4 tell you about age and weight for this person? (Answer in a complete sentence.) We can interpret f'(6) = 4 to mean that at six years of age, the person's weight is increasing at a rate of 4 pounds per year. Also, we could say that f'(6) = 4 tells us that we expect the person to gain about 4 pounds in the next year.
 - (c) What would f'(a) = 0 mean and is this reasonable? That is, would you ever expect this to happen? (Compete sentences, please.) Given our interpretation in part (b), we know that f'(a) = 0 tells us that at age *a* years, this person's weight is increasing at a rate of 0 pounds per year. That is, at this instant, he/she is not gaining weight. You certainly expect this to happen as someone ages. In particular, unless you expect a person to only GAIN weight throughout life, there MUST be some point(s) where f' is zero, even if only for an instant between gaining and losing (or losing and gaining) weight.

8. A rocket is launched vertically and is tracked by a radar station, which is located on the ground 3 miles from the launch site. What is the vertical speed of the rocket at the instant when its distance from the radar station is 5 miles and this distance is increasing at the rate of 5000 miles per hour? (Make sure to include units with your answer.)

Your picture here should be a right triangle with base of length 3 miles. (The distance from the launch pad to the radar station does not change with time.) The height of the triangle, we'll call A. The hypotenuse (which is also changing), we'll call C.

Let's translate the problem with our new variables. We are asked to find dA/dt when C = 5 and dC/dt = 5000.

The equation that relates our variables is: $A^2 + 3^2 = C^2$. Thus, when C = 5, we know A = 4. We take the derivative implicitly with respect to time to get:

2A(dA/dt) = 2C(dC/dt). Now we plug in numbers and solve for what we want to get: dA/dt = 8250 mi/hr.