October 26, 2007 Faudree

NAME:\_\_\_\_\_

There are 8 questions on this exam for a total of 100 points. You may not use calculators, books, or notes. You must show your work to receive full credit. Simplifying the derivatives is not required. However, if you do simplify, you must do so correctly. If you do not simplify, your answer must be correctly and unambiguously written. Answers to all questions must be clearly identified. The last page of the exam contains a 5 point extra credit problem and some derivative formulas.

You have one hour to complete the exam.

problem	points
1	
2	
3	
4	
5	
6	
7	
8	
total	

1. (4 parts for 10 points each) Find the derivative of each of the following:

(a) 
$$y = \sqrt[3]{x} - \frac{10}{x^2} + \ln x + e^2$$

(b)  $g(x) = 5\sin^{-1}(4x)$ .

(c)  $y = e^{-2x} \csc x$ 

(d)  $y = (x + \tan(3x))^8$ 

2. (10 points) Find dy/dx for

 $4xe^y = 5 + x^2 + y^3.$ 

3. (10 points) Find the derivative of the function  $y = (x^2 + 1)^{\sin x}$ .

4. (10 points) Use linearization or differentials to approximate  $(0.99)^7$ .

- 5. (10 points) The position of a particle is given by the equation  $s = 4t^2 16t + 20$  where t is measured in seconds and s is measured in feet.
  - (a) Find the velocity and acceleration of the particle.

(b) When is the particle moving to the right?

(c) What is the location(s) of the particle when it is at rest?

6. (5 points) Use the quotient rule to find the derivative of  $f(x) = \frac{3x^2 + \sqrt{2}}{\cos x}$ .

- 7. (5 points) The weight, W, in pounds, of a person is a function of the age, a, of the person in years. That is, we have W=f(a).
  - (a) What are the units of f'(a)?
  - (b) What does f'(8) = 4 tell you about age and weight for this person? (Answer in a complete sentence.)
  - (c) What would f'(a) = 0 mean and is this reasonable? That is, would you ever expect this to happen? (Compete sentences, please.)

8. (10 points) A rocket is launched vertically and is tracked by a radar station, which is located on the ground 3 miles from the launch site. What is the vertical speed of the rocket at the instant when its distance from the radar station is 5 miles and this distance is increasing at the rate of 5000 miles per hour? (Make sure to include units with your answer.)

## EXTRA CREDIT

Use logarithmic differentiation and implicit differentiation to prove that

$$\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2 - 1}}$$

## DERIVATIVES of INVERSE TRIGONOMETRIC FUNCTIONS

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}} \qquad \qquad \frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2} \qquad \qquad \frac{d}{dx}(\csc^{-1}(x)) = \frac{-1}{x\sqrt{x^2-1}}$$
$$\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2-1}} \qquad \qquad \frac{d}{dx}(\cot^{-1}(x)) = \frac{-1}{1+x^2}$$