

November 16, 2007  
Faudree

NAME: \_\_\_\_\_

There are 8 questions on this exam for a total of 100 points. You may not use calculators, books, or notes. You must show your work to receive full credit. There is an extra credit problem on the last page.

You have one hour to complete the exam.

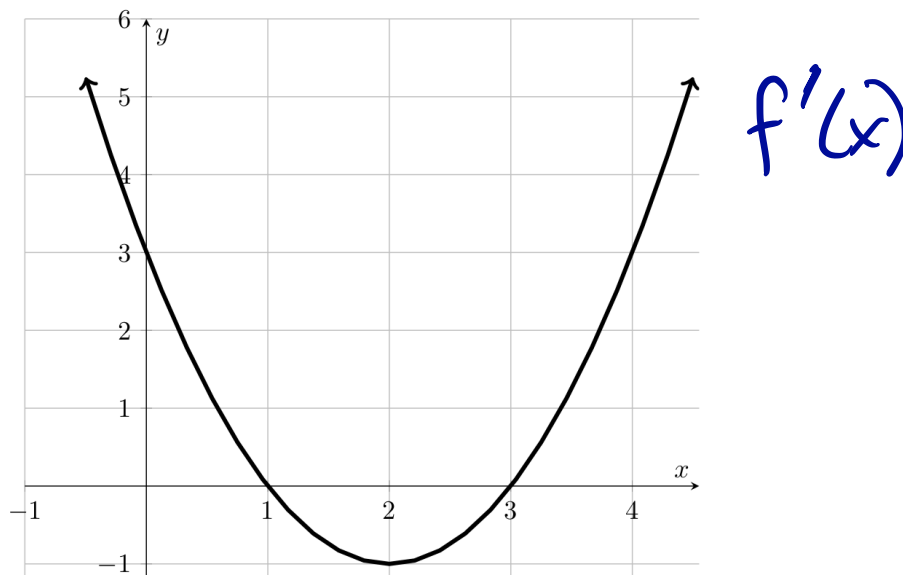
problem	points
1	
2	
3	
4	
5	
6	
7	
8	
total	

1. (10 points)

(a) Find the critical numbers of  $f(x) = x^3e^x$ .

(b) Apply the Second Derivative Test to the critical numbers of  $f(x)$  and describe what this test tells you about the behavior of  $f(x)$  at these points?

2. (10 points) Use the graph of  $f'(x)$  (the derivative), sketched below to answer the following questions. If there is not enough information to answer the question, answer "don't know."



- (a) On what interval(s) is  $f(x)$  increasing?
- (b) Does  $f(x)$  have a relative maximum?
- (c) Does  $f(x)$  have an absolute maximum?
- (d) On what interval(s) is  $f(x)$  concave up?
- (e) On what interval(s) is  $f(x)$  positive?

3. (10 points) Identify any local and absolute extreme values of the function  $f(x) = \frac{1}{4}x + \frac{1}{x}$  on the interval  $[1, 8]$ , if any exist. You must show your work.

local minimum(s): \_\_\_\_\_ local maximum(s): \_\_\_\_\_

absolute minimum(s): \_\_\_\_\_ absolute maximum(s): \_\_\_\_\_

4. (10 points each) Evaluate the limits below. Give the most complete answer possible. Show your work.

(a)  $\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}$

(b)  $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^{3x}$

5. (15 points) Let  $f(x) = \frac{5x^2-1}{\sqrt{x^4-1}}$ .

(a) Find the domain of  $f(x)$ .

(b) Find any  $x$ -intercepts for  $f(x)$ .

(c) Find any  $y$ -intercepts for  $f(x)$ .

(d) Find any horizontal asymptotes and show your answer is correct.

6. (15 points) Given  $f(x) = 4x^{1/3} + x^{4/3}$ . Answer the questions below about the graph of  $f(x)$ .

NOTE:  $f'(x) = \frac{4(x+1)}{3x^{2/3}}$  and  $f''(x) = \frac{4(x-2)}{9x^{5/3}}$

(a) Determine the intervals where  $f$  is increasing and where  $f$  is decreasing.

(b) Determine the intervals where  $f$  is concave up and where  $f$  is concave down.

(c) Identify any local maximums or minimums.

(d) Identify any inflection points.

7. (10 points) Clearly and neatly sketch ONE graph containing ALL the properties below.

- (a)  $\lim_{x \rightarrow \infty} f(x) = 2$  and  $\lim_{x \rightarrow 1^-} f(x) = \infty$
- (b)  $x$ -intercepts at 0 and 2
- (c)  $f' > 0$  on  $(-\infty, 1) \cup (1, 3)$  and  $f' < 0$  on  $(3, \infty)$
- (d)  $f'' > 0$  on  $(-\infty, 1) \cup (4, \infty)$  and  $f'' < 0$  on  $(1, 4)$



8. (10 points) A cylindrical tank is to be constructed. The cost of construction per square unit of surface area is four times as much for the top and bottom of the tank as for the sides. Determine the dimensions to be used if the volume is fixed and the cost of construction is to be kept to a minimum. You must show your work. Make sure to clearly define your variables, identify the domain of your function, show (using calculus) that your answer is correct, and completely answer the question.

## EXTRA CREDIT

A marathoner ran the 26.2 mile New York City Marathon in 2.2 hours. Use the Mean Value Theorem to prove that at least twice, the marathoner was running precisely 11 mph. You MUST use the Theorem explicitly to receive credit. That is, you must carefully identify your  $f(x)$ , make sure the MVT applies, etc. Hint: You will need another important theorem too...

Mean Value Theorem: Let  $f(x)$  be a function that is continuous on  $[a, b]$  and differentiable on  $(a, b)$ , then there is a number  $c$  in  $(a, b)$  such that  $f'(c) = \frac{f(b)-f(a)}{b-a}$ .