October 5, 2007
Faudree NAME:_____

There are 9 questions on this exam for a total of 100 points. You may not use calculators, books, or notes. You must show your work to receive full credit. There is a 5 point extra credit problem at the end of the exam. You have one hour to complete the exam.

1. Let $f(x) = \sqrt[4]{1-x}$.

   (a) (6 points) Find $f^{-1}(x)$.

   (b) (4 points) Determine the domain and range of $f^{-1}(x)$.

2. (5 points each) Evaluate the limits below, if possible. Give the most complete answer.

(a) $\lim_{x \to \sqrt{3}} \dfrac{2 + \cos x}{x}$

(b) $\lim_{x \to 2^-} \dfrac{4 - x^2}{2 - x - x^2}$

(c) $\lim_{x \to -3^+} \dfrac{1 + e^x}{3 + x}$

3. (5 points each) Evaluate the following limits at infinity, if possible. Give the most complete answer.

   (a) $\lim_{t\to\infty} \dfrac{5t - 3t^2}{\sqrt{\pi + 2t^4}}$

   (b) $\lim_{y\to\infty} \ln(1 + \dfrac{1}{y})$

4. (5 points) For what x-values, if any, does the function $H(x) = \dfrac{5}{2e^{x-1} - 3}$ fail to be defined?

5. (10 points) Sketch the graph of $y = 2\sin(x+1)$. Label any x- or y-intercepts and label high and low points on the curve. I recommend you sketch it in parts. That is, first sketch and label $y = \sin x$. Next, sketch and label $y = \sin(x+1)$. Then, sketch and label your final answer. This is a way of ensuring some partial credit even if your final answer is wrong.

6. (a) (5 points) Complete the definition of continuity.

   A function f is continuous at a number a if

   (b) (5 points) Given $f(x) = \begin{cases} 2 + \cos x & x < 0 \\ \frac{3}{1+x^2} & 0 \leq x \end{cases}$, use the definition of continuity to show that $f(x)$ is continuous at $x = 0$. You must show your work.

   (c) (5 points) Does the graph of $f(x)$ from part (b) have any horizontal asymptotes? Explain your answer in detail.

7. (a) (5 points) Complete the definition of the definition of the derivative.

   The derivative of a function f is defined as: $f'(x) =$

   (b) (15 points) Use the definition of the derivative to find the derivative of $g(x) = 3x + \frac{1}{x}$.

8. (10 points) Assume $f(x) = e^x + x + 2$ and $f'(x) = e^x + 1$. Find the equation of the line tangent to the graph of $f(x)$ at $x = 0$.

9. (5 points) Write the area of a square, A, as a function of its perimeter, P.

EXTRA CREDIT (5 points)
Use the definition of the derivative to prove the Chapter 3 Section 1 derivative rule below:

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

where c is a fixed constant.