1. Let $f(x) = \sqrt[4]{1-x}$.

   (a) I will apply the usual method. We start with $y = \sqrt[4]{1-x}$ and switch x and y to get: $x = \sqrt[4]{1-y}$. Now solve for y to get $y = 1 - x^4$. So, $f^{-1}(x) = 1 - x^4$.

   (b) Determine the domain and range of $f^{-1}(x)$.
   For me it is easier to look at f.
   domain of f : We need $1 - x \geq 0$. So $1 \geq x$. So the

   domain of f= range of $f^{-1} = (-\infty, 1]$.

   range of f: Since the function is defined as the *positive* square root, we get all real numbers greater than or equal to zero. So the

   range of f = domain of $f^{-1} = [0, \infty)$.

   Note that the domain of f^{-1} CANNOT be $(-\infty, \infty)$ as it would not, in that case, have an inverse.

2. Evaluate the limits below, if possible. Give the most complete answer.

   (a) $\displaystyle\lim_{x\to\sqrt{3}} \frac{2+\cos x}{x} = \frac{2+\cos\sqrt{3}}{\sqrt{3}}$
   (The method is to plug in $\sqrt{3}$ for x since all the "pieces" are continuous.)

   (b) $\displaystyle\lim_{x\to 2^-} \frac{4-x^2}{2+x-x^2} = \lim_{x\to 2^-} \frac{(2-x)(2+x)}{(2-x)(1+x)} = \lim_{x\to 2^-} \frac{2+x}{1+x} = 4/3$

   (c) $\displaystyle\lim_{x\to -3^+} \frac{1+e^x}{3+x} = \infty$
   Reasoning: We observe that $\lim_{x\to -3}(1+e^x) = 1+e^{-3} > 0$ and $\lim_{x\to -3^+}(3+x) = 0$ and always positive. Thus the quotient approaches positive infinity.

3. Evaluate the following limits at infinity, if possible. Give the most complete answer.

   (a) $\displaystyle\lim_{t\to\infty} \frac{5t-3t^2}{\sqrt{\pi+2t^4}} \cdot \frac{\frac{1}{t^2}}{\frac{1}{t^2}} = \lim_{t\to\infty} \frac{\frac{5}{t}-3}{\sqrt{\frac{\pi}{t^4}+2}} = -3/\sqrt{2}$

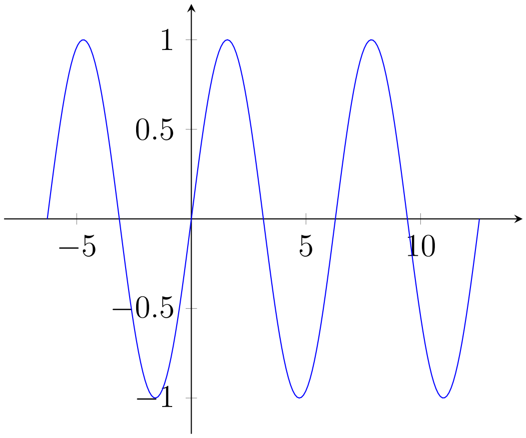
   (b) $\displaystyle\lim_{y\to\infty} \ln(1+\frac{1}{y}) = \ln(1) = 0$
   Reasoning: We observe that $\lim_{y\to\infty} 1+\frac{1}{y} = 1$. Thus, $\lim_{y\to\infty}\ln(1+\frac{1}{y}) = \lim_{x\to 1}\ln x = \ln 1 = 0$.

4. For what x-values, if any, does the function $H(x) = \dfrac{5}{2e^{x-1}-3}$ fail to be defined?
   We must avoid $2e^{(x-1)} - 3 = 0$. Now, solve this for x : $e^{(x-1)} = 3/2$. So, $x - 1 = \ln(3/2)$. Finally: $x = 1 + \ln(3/2)$.

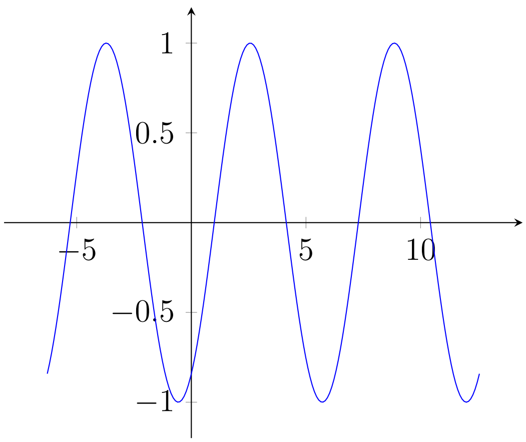
5. Sketch the graph of $y = 2\sin(x+1)$.
   The graph looks like $y = \sin(x)$ but is shifted left 1 unit and stretched by 2 units vertically. Thus, the x-intercepts are: $..., -\pi - 1, -1, \pi - 1, 2\pi - 1, 3\pi - 1....$ The y-intercept is $2\sin(1)$. A high point is $(\pi/2 - 1, 2)$, a low point is $(3\pi/2 - 1, -2)$. You

can check this on your calculator.

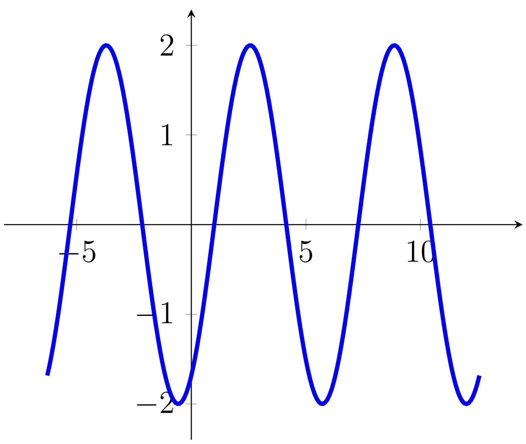
$y = \sin x$



$y = \sin(x - 1)$



$y = 2\sin(x - 1)$



6. (a) Complete the definition of continuity.

A function f is continuous at a number a if (see your text!)

(b) Given $f(x) = \begin{cases} 2+\cos x & x<0 \\ \frac{3}{1+x^2} & 0 \le x \end{cases}$, use the definition of continuity to show that $f(x)$ is continuous at $x=0$. You must show your work.
We will have to find $f(0)$ and $\lim_{x\to 0} f(x)$ and make sure they are equal.
So $f(0) = 3/(1+0^2) = 3$.
We need the two-sided limit. But since the function is defined in pieces, we will have to check each side separately and hope they are the same.
So, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(2+\cos x) = 2+\cos 0 = 3$.
And, $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(3/(1+x^2)) = 3/(1+0^2) = 3$.
Finally, since the two one-sided limits are in fact the same, we can conclude :
$\lim_{x\to 0} f(x) = 3 = f(0)$ and f is indeed continuous at $x=0$.

(c) Does the graph of $f(x)$ from part (b) have any horizontal asymptotes? Explain your answer in detail.
Here we have to check the limit of f as x approaches ∞ and $-\infty$.
So,
$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{3}{1+x^2} = 0$. So, $y=0$ is an asymptote.
$\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} 2+\cos x$. Now this limit does not exists because the cosine function oscillates. So there are no other asymptotes.

7. (a) Complete the definition of the definition of the derivative.

The derivative of a function f is defined as: $f'(x) =$ (see your text)

(b) Use the definition of the derivative to find the derivative of $g(x) = 3x + \frac{1}{x}$.

$$g'(x) = \lim_{h\to 0} \frac{(3(x+h)+\frac{1}{x+h}) - (3x+\frac{1}{x})}{h} = \lim_{h\to 0} \frac{3x+3h+\frac{1}{x+h}-3x-\frac{1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{3h+\frac{1}{x+h}-\frac{1}{x}}{h} = \lim_{h\to 0} \frac{3h}{h} + \lim_{x\to 0} \frac{1}{h} \cdot \frac{x-(x+h)}{x(x+h)}$$

$$= \lim_{h\to 0} 3 + \lim_{h\to 0} \frac{1}{h} \cdot \frac{-h}{x(x+h)} = 3 + \lim_{h\to 0} \frac{-1}{x(x+h)} = 3 - \frac{1}{x^2}$$

8. Assume $f(x) = e^x + x + 2$ and $f'(x) = e^x + 1$. Find the equation of the line tangent to the graph of $f(x)$ at $x=0$.
We need a point on the line and the slope of the line.
Point: $f(0) = e^0 + 0 + 2 = 3$. So the point $(0,3)$ is on the line.
Slope: $m = f'(0) = e^0 + 1 = 2$.
Finally, the equation of the line is: $y - 3 = 2(x-0)$ or $y = 2x+3$.

9. Write the area of a square, A, as a function of its perimeter, P.
So we need an equation that has A on one side and the other has P's and numbers and nothing else.

The standard equations for A and P are in terms of the side of the square, s. That is: $A = s^2$ and $P = 4s$.
So, $s = P/4$ which we plug into the first equation to get: $A = (p/4)^2$, the answer.

EXTRA CREDIT
Use the definition of the derivative to prove the Chapter 3 Section 1 derivative rule below:

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

where c is a fixed constant.
This argument is derived in section 3.2. And thank you to the two students who got it perfectly right.